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一种新的三维位势问题多极边界元离散方法

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摘 要:应用快速多极展开法将三维位势问题的边界方程离散. 由于在边界方程基本解中,含有奇异项$1/r$,影响了多极展开(FMM)的应用,笔者利用拉普拉斯变换,可以转换为指数形式序列. 这种方法避免了多极展开的奇异性,得到了新的离散解析式,为理论分析多极边界元提供了一种新的方法.

关键词:快速多极展开;拉普拉斯变换;位势问题;基本解

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0 引言

快速多极展开法(FMM)是对求和项$\sum_{i=1}^{n}\frac{1}{r_i}$的近似计算方法,计算精确程度和展开的指数项数目密切相关. 20 世纪 80 年代后期,Greengard 等[1]提出一种快速多极展开法,基于球面谐函数在空间中的多极展开,采用递归算法结构使计算量级降为 O(n)阶. FMM 是用迭代法解决边界积分方程,广泛的应用于线性弹性问题和流体力学. 近几年,国内学者对 FMM 的应用给予了积极的关注. 2002 年,姚振汉教授课题组率先将泰勒级数展开式用于二维线弹性问题边界元法中[2],并且不断在二维、三维问题中进行完善[3-4]. 与此同时,申光宪课题组将静电多极展开式用于三维线弹性的边界积分方程[5]. 这两种方法有一个共同的特点:离散之后的边界方程,都含有奇异项 $1/r$. 为了克服上述的奇异性,笔者首先在快速多极展开法的基础上,将三维位势问题的基本解分解,然后用拉普拉斯(Laplace)变换,将奇异项 $1/r$ 转换为指数级数形式,因此在计算其偏导的时候不出现奇异项.

1 拉普拉斯变换

设$f(t)$定义在$[0,+\infty)$,如果存在积分

$$F(p)=\int_0^{\infty} f(t)e^{-pt}\mathrm{d}t,\ (p=\xi+i\eta) \quad (1)$$

$F(p)$就是$f(t)$的拉普拉斯变换[6]. 令$f(t)=1$,$p=r$(r是实数),则有

$$\frac{1}{r}=\int_0^{\infty} e^{-t_\theta r}\mathrm{d}t=\sum_{\theta=0}^{q} w_\theta e^{-t_\theta r}$$

式中:ω_θ是权系数.

2 新的三维位势问题多极边界元离散

设控制方程为 $\Delta^2 u=0$ 在 Ω 内,基本边界条件:$u=\bar{u}$ 在 Γ_1 上;自然边界条件:$q=\frac{\partial u}{\partial n}=\bar{q}$ 在 Γ_2 上. 其中 $\boldsymbol{n}$ 为边界 Γ 的外法矢,$\Gamma=\Gamma_1+\Gamma_2$,字母上方一杠表示已知. 利用 Green 公式可得边界积分方程[7]为

$$c_{ij}(x)u^i(x)=\int_\Gamma u^*(x,t)q(y)\mathrm{d}\Gamma-\int_\Gamma q^*(x,y)u(y)\mathrm{d}\Gamma \quad (2)$$

其基本解为

$$u^*=\frac{1}{4\pi r} \quad (3)$$

$$q^*=\frac{\partial u^*}{\partial n}=\frac{\partial}{\partial n}\left(\frac{1}{4\pi r}\right) \quad (4)$$

式中:c_{ij}为边界形状系数,Γ 为表面边界. x 为源点,y 为边界 Γ 上任意一点.

$$r=|x-y|$$

$\boldsymbol{n}$ 为边界 Γ 的外法矢.

若边界 Γ 上的单元坐标表示为:$x_i^k(\xi)=\sum_l x^{kl}\phi^l(\xi)$,$\xi$ 是单元的局部坐标,x^{kl}是 k 单元 l

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节点的形函数,则对于等参元可以得到[8-9]

$$u_i^k[x(\xi)] = \sum_l u_i^{kl}\phi^l(\xi) \tag{5}$$

$$t_i^k[x(\xi)] = \sum_l t_i^{kl}\phi^l(\xi) \tag{6}$$

利用式(5)和(6),在点 x^q 对式(2)数值积分得:

$$c^i u^i(x^q) = -\sum_{k,l,s} q^*[x^q,y(\xi^s)]u^{kl}\phi^l(\xi^s)J[y(\xi^s)]\omega^s + \sum_{k,l,s} u^*[x^q,y(\xi^s)]q^{kl}\phi^l(\xi^s)J[y(\xi^s)]\omega^2 \tag{7}$$

由(7)式可知,对每一个单元区域上的运算 $\sum_{l,s}\phi^l(\xi^s)J|y(\xi^s)|\omega^s$,在所有边界点对其积分时是不变的,如果采用迭代的方法求解方程,u^{kl} 和 q^{kl} 在迭代前赋值,那么在每一步迭代中,乘积项

$$\sum_{k,l,s} u^{kl}\phi^l(\xi^s)J|y(\xi^s)|\omega^s$$

对各个单元是定值,同理乘积项

$$\sum_{k,l,s} q^{kl}\phi^l(\xi^s)J|y(\xi^s)|\omega^s$$

也有同样的性质. 此时若 $q^*[x^q,y(\xi^s)]$ 和 $u^*[x^q,y(\xi^s)]$ 可以写成 $\frac{1}{r}$ 的函数,即

$$q^*[x^q,y(\xi^s)] = f_1(x^q)\left(\frac{1}{r}\right),$$

$$u^*[x^q,y(\xi^s)] = f_2(x^q)\left(\frac{1}{r}\right).$$

则,式(7)可以用快速多极展开计算. 实际上

$$u^* = \frac{1}{4\pi r} = f_1(x)\left(\frac{1}{r}\right),$$

$$q^* = \frac{\partial u^*}{\partial n} = \frac{\partial}{\partial n}\left(\frac{1}{4\pi r}\right) = \frac{1}{4\pi}\left[\partial_1\left(\frac{1}{r}\right)n_1 + \partial_2\left(\frac{1}{r}\right)n_2 + \partial_3\left(\frac{1}{r}\right)n_3\right] = \frac{1}{4\pi}(\partial_1 n_1 + \partial_2 n_2 + \partial_3 n_3)\left(\frac{1}{r}\right) = f_2(x)\left(\frac{1}{r}\right)$$

因此,式(7)可以写成为

$$\begin{aligned} c^i u^i(x^q) = & -\sum_{k,l,s} q^*[x^q,y(\xi^s)]u^{kl}\phi^l(\xi^s)J|y(\xi^s)|\omega^s + \sum_{k,l,s} u^*[x^q,y(\xi^s)]q^{kl}\phi^l(\xi^s)J|y(\xi^s)|\omega^s = \\ & -\sum_{k,l,s} f_2(x^q)\left(\frac{1}{r}\right)u^{kl}\phi^l(\xi^s)J|y(\xi^s)|\omega^s + \sum_{k,l,s} f_1(x^q)\left(\frac{1}{r}\right)u^{kl}\phi^l(\xi^s)J|y(\xi^s)|\omega^s \end{aligned} \tag{8}$$

从式(8)可以看到,我们得到含有奇异项 $\frac{1}{r}$ 的离散式,此时在对上式右侧奇异项进行拉普拉斯变换,得到新的离散式如下:

$$\begin{aligned} c_{ij}u^i(x^q) = & -\sum_{\theta=0}^{q} w_\theta \sum_{k,l,s} f_2(e^{-t\theta})u^{kl}\phi^l(\xi^s)J[y(\xi^s)]\omega^s + \sum_{\theta=0}^{q} w_\theta \sum_{k,l,s} f_1(e^{-t\theta})u^{kl}\phi^l(\xi^s)J[y(\xi^s)]\omega^s = \\ & -\sum_{k,l,s,\theta} w_\theta f_2(e^{-t\theta})u^{kl}\phi^l(\xi^s)J[y(\xi^s)]\omega^s + \sum_{k,l,s,\theta} w_\theta(e^{-t\theta})u^{kl}\phi^l(\xi^s)J[y(\xi^s)]\omega^s \end{aligned} \tag{9}$$

3 结论

从(9)式可以看出对边界积分方程(2)进行了离散处理,将其转换为指数级数形式,消除了奇异项 $\frac{1}{r}$ 的影响. 如何处理奇异项是边界元方法中很重要的一个问题,笔者在理论上为处理多极边界元的奇异性,提供了一种新的离散处理方法.

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Treatment of Vehicle Load and Axle Load Model

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Abstract: Base on analysis of axle load model of vehicle, coefficient of axle load application is set up by this research, the concept has ability what illustrate affect of mechanics and characteristic of traffic more clearly; the data of lorry's axle load as sample, and statistic as tool, the new model of axle load is set up by the paper too. It is shown that weather normal or heavy traffic, lorry 's axle load of single wheel which single axle, double wheels which single axle, double axles and tri－axles styles under loading and unloading get close to Gauss' Distribution. Axle structure, loading and unloading of vehicle are parameters of the model. The model has advantages what disturb of vehicle style is avoided, the calculation's error which caused by the difference between theory load axle and actual load axle is avoided too, at the same time, it make investigation of traffic easy, estimation of road more well and usage in engineering convenient.

Key words: coefficient of axle load application axle load model Gauss' Distribution axle structure loading vehicle unloading vehicle

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A New Kind of Discrete Method with FM-BEM about 3D Potential Problem

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Abstract: By applying fast multipole expansion, the boundary integral equation about 3D potential problem is made discrete. In boundary, because basic solutions contain singular term , which influences the application of multipole expansion method, but by Laplace transformation it can be reduced to exponential series. Then this method avoids the originaLity of FMM, gets the new discrete equality and provides a new method for theory analysis of FMM.

Key words: fast multipole expansion; Laplace transformation; potential problem; basic solution