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Discrete Maximum Principle Model on Controlling Urban Residential Market

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Abstract: Aiming at controlling supply and demand of urban residential market and minimizing its loss for unbalance, This paper, from the angle of storage management, combining with its dynamic balanced characteristics of supply and demand, sets up discrete maximum principle model of controlling storage of urban residential market with the storage theory in economic control theory. Annual quantities of demand in analytic period given, this model can provide annual optimum quantities of supply and storage in analytic period to minimize the market losses. Sample analytic has been made for urban residential market of the country by the model. Through verifications, this model has important application values in reasonable planning annual supply and storage of urban residential market in analytic period and getting the optimum results.

Key words: urban residence; storage; maximum principle

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Introduction

At present, owing to all kinds of reasons involving its internal development and external residential system, there are serious problems for storage management in urban residential market of China. The prominent performances are serious unbalance between supply and demand and too much overstocking of residence. What resulted in the above problems is that insufficient attention was paid to controlling urban residential market and control method was unwise.

The optimum analysis of general market aims at the balance between supply and demand, whereas residential market has a longer cycle and its supply and demand are obviously relatively inactive and stagnant. Which necessitates the existences of dynamic balanced characteristics and storage of residential market[1~3]. So, it doesn't correspond with the realities of balancing residential market precisely that the balance between supply and demand is regarded as the optimum. The necessity of residential storage necessitates the losses from unbalance between supply and demand. The authors think that the optimum urban residential market should be based on the rational storage for minimizing the losses.

1 Description of the State Space of Supply and Demand and Controlling Storage of Urban Residential Market

General speaking, the unbalance between supply and demand of urban residential market results in two kinds of main losses.

(1) The more the square of the difference between supply and demand of urban residential market, the moreeconomical losses are. If supply is smaller than demand, there will be the loss for insufficient residence; if supply is 1arger, there is the loss for selling too much residence.

(2) The larger square of the difference between actual storage and ideal one of urban residence, the more losses undertaken. If actual storage is larger than ideal one, extra money for storage will be paid; if actual

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storage is smaller, the risk loss resulting from insufficient storage will be undertaken.

According to the analysis above, we assume the quantity of supply S_t as controlling variable of market system, the quantity of demand D_t as internal variable of the system, the quantity of storage x_t as the state variable. Then, the state equation of control system on urban residential market as fellow:

$$\frac{x_{t+1} - x_t}{\Delta t} = S_{t+1} - D_{t+1}. \tag{1}$$

We define the loss for unbalance between supply and demand $I\ (x, S)$ as objective variable and the output variable of the system. The output equation of the system is:

$$I(x,S) = \sum_{t=0}^{T-1} [C_1(S_t - D_t)^2 + C_2(x_t - \bar{x}_t)^2]\Delta t , \tag{2}$$

Where C_1 and C_2 are weight coefficients of two kinds of losses respectively.

The state equation and output equation constituent the description of state space on the storage control, supply and demand of urban residential market.

2 Discrete Maximum Principle Model on Storage Control for Urban Residential Market

When the time series S of control variable on urban residential market deviate the optimum one S^*, panto – function $I\ (x, S)$ will increase; whereas the objective panto – function $I(x^*, S^*)$ will make I near the minimal, when the loss of the system reach the minimal value. According to the state space description of the system above (see Equ. (1) & (2)), the urban residential market system will be regarded as a time – based and discrete one which has a local equation restrains $\left(\frac{x_{t+1} - x_t}{\Delta t} = S_{t+1} - D_{t+1}\right)$ and free right endpoint (x_T unknown). So the discrete maximum principle can be used to solve objective panto – function on storage control of urban residential market.

$$I(x,S)_{\min} = \sum_{t=0}^{T-1} [C_1(S_t - D_t)^2 + C_2(x_t - \bar{x}_t)^2]\Delta t , \tag{3}$$

subjected

$$\frac{x_{t+1} - x_t}{\Delta t} = S_{t+1} - D_{t+1}, \tag{4}$$

Where original value x_0 of time series x_t given, by means of Hamilton function, we get:

$$H = \sum_{t=0}^{T-1} \{[C_1(S_t - D_t)^2 + C_2(x_t - \bar{x}_t)^2]\Delta t + \lambda_t(S_{t+1} - D_{t+1})\} . \tag{5}$$

According to discrete maximum principle:

$$\begin{cases} -\frac{\partial H}{\partial x_t} = \frac{\lambda_{t+1} - \lambda_t}{\Delta t} = -2C_2(x_{t+1} - \bar{x}_{t+1}) ; \\ \frac{\partial H}{\partial \lambda_t} = \frac{x_{t+1} - x_t}{\Delta t} = S_{t+1} - D_{t+1} ; \\ \frac{\partial H}{\partial S_t} = 2C_1(S_{t+1} - D_{t+1}) + \lambda_t = 0 , \end{cases} \tag{6}$$

Solving eauations above:

$$S = D - \frac{\lambda}{2C} ; \frac{x_{t+1} - x_t}{\Delta t} = \frac{-\lambda_t}{2C_1} ;$$

$$\frac{\lambda_{t+1} - \lambda_t}{\Delta t} = 2C_2(x_{t+1} - \bar{x}_t + 1) .$$

Generally speaking, ideal storage value $\bar{x}_t = 0$, $t = 0, 1, 2, \cdots, T-1$. Where assuming $\Delta t = 1$, the latter two expressions being converted to the system of equation of difference as fellow:

$$\begin{cases} x_{t+1} = x_t - \frac{1}{2C_1}\lambda_t ; \\ \lambda_{t+1} = \lambda_t - 2C_2 x_{t+1}. \end{cases} \tag{7}$$

Putting the second expression of Equ. (7) into the first one, we can get the expression of the optimum storage x_t of t year as fellow:

$$x_t = \left(1 + \frac{C_2}{C_1}\right)x_{t-1} + \frac{C_2}{C_1}x_{t-2} + \frac{C_2}{C_1}x_{t-3} + \Lambda + \frac{C_2}{C_1} - \frac{1}{2C_1}\lambda_0. \tag{8}$$

From Equ. (7) the differential equation on x given:

$$x_{t+1} - \left(2 + \frac{C_2}{C_1}\right)x_t + x_{t-1} = 0 . \tag{9}$$

Likewise, the differential equation on λ_t given:

$$\lambda_{t+1} - \left(2 + \frac{C_2}{C_1}\right)\lambda_t\lambda_{t-1} = 0 . \tag{10}$$

Combining with given $x_0 = a_0$ and Equ. (9) and (10), we can get discrete maximum principle model on storage control of urban residential market with classical solution of differential equations:

The optimum supply plan of urban residential market is:

$$S_t = D_t - \frac{B_1\alpha_1^t + B_2\alpha_2^t}{2C_1} . \tag{11}$$

The optimum storage control plan is

$$x_t = A_1\alpha_1^t + A_2\alpha_2^t. \tag{12}$$

Where,

$$\alpha_1 = \frac{2C_1 + C_2 + \sqrt{C_2(4C_1 + C_2)}}{2C_1}; \tag{13}$$

$$\alpha_2 = \frac{2C_1 + C_2 - \sqrt{C_2(4C_1 + C_2)}}{2C_1}; \tag{14}$$

$$A_1 = \frac{\alpha_2^T(\alpha_2 - 1)\alpha_0}{\alpha_1^{T+1} - \alpha_1^T - \alpha_2^{T+1} + \alpha_2^T}; \tag{15}$$

$$A_2 = \frac{\alpha_1^T(\alpha_1 - 1)\alpha_0}{\alpha_1^{T+1} - \alpha_1^T - \alpha_2^{T+1} + \alpha_2^T}; \tag{16}$$

$$B_1 = \frac{2\alpha_1^T C_2\alpha_0}{-\alpha_1^{T+1} + \alpha_1^T + \alpha_2^{T+1} - \alpha_2^T}; \tag{17}$$

$$B_2 = \frac{-2\alpha_1^T C_2\alpha_0}{-\alpha_1^{T+1} + \alpha_1^T + \alpha_2^{T+1} - \alpha_2^T}; \tag{18}$$

Where values of D_t, C_1, C_2, a, T substituted; we can get time series of the optimum supply quantity and the optimum control storage. The time series are the optimum ones on urban residential market which the discrete maximum principle is applied.

3 The Grey System Predicting and Controlling of Urban Residetial Market of the Whole Country

Generally speaking, the unbalanced coefficients of supply and demand C_1 and unbalanced coefficients of storage on urban residential market of the whole country can be selected as 0.5 respectively, the ideal annual storage quantity $x = 0$. With the model above, we can get the expressions of the optimum annual supply model and storage model on urban residential market of the whole country. The expressions determined as follow: The optimum control results on urban residential market of the whole country:

(1) The optimum storage plan

$$x_t = 0.01717 \times 2.62^t + 4661.98 \times 0.38^t \quad (t = 1 \sim 6); \tag{19}$$

(2) The optimum shadow price

$$\lambda_t = -0.02779 \times 2.62^t + 2881.26 \times 0.38^t \quad (t = 1 \sim 6); \tag{20}$$

(3) The optimum supply plan

$$S_{t+1} = D_{t+1} + 0.02779 \times 2.62^t - 2881.26 \times 0.38^t \quad (t = 0 \sim 5). \tag{21}$$

Using our software named Grey System Predicting and Controlling System of Urban Residential Market (GSPCURM)[4] in which mathematics method of control part adopts the discrete maximum principle model, table 1 gives the optimum control results of on urban residential market of the whole country[5]. Results of table 1 as fellow.

Table 1　The optimum control results of on urban residential market of the whole country from 1990 to 2002

Years	Q.A. D.Q. T.M.	P.A. D.Q. T.M.	Q.A. S.Q. T.M.	Q.A. ST.Q. T.M.	Q.A. C.L. T.M.	O.OP.A. S.Q. T.M.	O.OP.A. ST.Q. T.M.	O.OP.A. LL.C.L. T.M.	OP.P.A. S.Q. T.M.	OP.P.A. ST.Q. T.M.	P.LL. C.L. T.M.
1990	2545	2545	2631	277	4.2E+4	2427	73	9.23E+3	–	–	–
1991	2745	2769	3124	656	3.3E+5	2700	28	1.10E+4	–	–	–
1992	3818	4037	4313	1151	1.1E+6	3801	11	1.12E+4	–	–	–
1993	6035	6087	6505	1621	2.5E+6	6028	4	1.13E+4	–	–	–
1994	6118	6124	7497	3000	8.0E+6	6116	2	1.13E+4	–	–	–
1995	6787	6203	7549	3762	1.5E+7	6706	1	1.13E+4	–	–	–
1996	6896	6894	7886	4662	2.7E+7	6896	1	1.13E+4	–	–	–
1997	–	7003	–	–	–	–	–	–	4122	1781	5.78E+6
1998	–	7137	–	–	–	–	–	–	6036	680	6.61E+6
1999	–	7267	–	–	–	–	–	–	6847	260	6.74E+6
2000	–	7407	–	–	–	–	–	–	7247	100	6.75E+6
2001	–	7559	–	–	–	–	–	–	7499	40	6.76E+6
2002	–	7726	–	–	–	–	–	–	7706	20	6.76E+6

Note: owing to limited space of the first rank of table, we assume that: O. represents original, likewise, OP. represents optimum, P. represents predicting, C. represents cumulative, L. represents loss, A. represents annual, S. represents supply, D. represents demand, Q. represents quantity, T.M. represents 10 thousand and square meter, LL represents lowest, ST. represents storage. Thus combination of these letters above represent a phrase. For example, O.A.S.Q. represent original annual demand quantity.

From table 1, respective cumulative annual of the optimum values of supply and demand, compared with relative cumulative values of the original supply and demand, induce by 2 ~ 3 quantity grade. Which verifies the accuracy of the predictive and controlling model on the other hand.

Suggestions on control of residential market of our country from 1990 to 2002 from table are: we should try our best to reduce the residential storage dramatically to 18 million m^2, make the residential supply at the level of about 59 million m^2 by the late of 1997; so that we can get rid of storage basically by the late of 2000 when the residential supply approach the residential demand. Through the suggesting methods above, the whole urban residential market of our country will reach the optimum globally from 1990 to 2002.

4 Conclusion

At present, the control strength should be reinforced on residential market, from the angle of storage management, to make urban residential market of our country reach the optimum. This paper set up the discrete maximum principle model on urban residential market with relative theory and reasoned the optimum solution of the model. By means of the model above, sample analysis has been done to verify its accuracy and practicality. So the model has widely application values in heightening officials' macro control capacities on urban residential market and managers' levels of the investment, management and decision.

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城市住宅市场调控的离散型最大值原理模型

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摘　要：从城市住宅市场库存管理的角度，结合城市住宅市场供求平衡的动态特性，以调控城市住宅市场供求，保证市场供需失衡损失最小化为目的，利用经济控制论的库存理论，建立了城市住宅市场库存调控的离散型最大值原理模型.该模型可在分析期内各年市场需求量已知的情况下，以市场损失为最小地给出分析期内各年的住宅最优供给量和最优库存量.利用模型对全国城市住宅市场进行了实例分析.经验证，该模型对于合理规划城市住宅市场在分析期内各年的供给和库存，从而达到市场最优有重要的应用价值.

关键词：城市住宅；库存；最大值原理