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惯性项对动静压浮环径向轴承压力场的影响

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摘 要: 利用基本简化的 Navier-Stocks 方程组, 获得无惯性力流体动压润滑轴承的流速场与压力场, 以其为近似初始条件, 导出在迁移惯性力影响下的非定常雷诺方程, 并在此基础上, 利用有限元方法, 求解非定常雷诺方程, 获得浮环轴承内外膜的的压力场, 结果表明: 惯性力对轴承的压力分布有明显的影。

关键词: 滑动轴承; 浮环; 惯性项;

中图分类号: TH 117.2 **文献标识码:** A

0 引言

动静压浮环轴承具有良好的稳定性能, 特别适合于高速运转工况, 有着良好的发展前景。一般情况下, 研究其性能仅从简化的基本雷诺方程出发, 但随着转速的提高, 流体惯性力的影响会逐步增加。本文在导出含惯性项的动静压浮环轴承内外膜雷诺方程的基础上, 应用有限元法讨论了惯性项对压力分布的影响。

1 圆柱坐标系下的雷诺方程

对直角坐标系的 Navier-Stocks 方程组^[1], 在薄膜润滑的情况下, 忽略当地惯性项, 计入迁移惯性项^[2], 便得到

$$\begin{cases} \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}; \\ \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \mu \frac{\partial^2 v}{\partial y^2}; \\ \rho \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial p}{\partial z} + \mu \frac{\partial^2 w}{\partial y^2}, \end{cases} \quad (1)$$

速度边界条件为

$$\begin{cases} y = 0 \text{ 时}, u = u_1, v = v_1, w = w_1 = 0; \\ y = h \text{ 时}, u = u_2, v = v_2, w = w_2 = 0. \end{cases}$$

令式(1)中左边为零, 得到无惯性影响的速度场

$$u = \frac{1}{2\mu} \frac{\partial P}{\partial x} y(y-h) + \frac{(u_2 - u_1)y}{h} + u_1, \quad (2)$$

$$v = \frac{(v_2 - v_1)y}{h} + v_1, \quad (3)$$

$$w = \frac{1}{2\mu} \frac{\partial P}{\partial z} y(y-h), \quad (4)$$

式中: v_1, v_2 为小扰动参量, 把 u, v, w 代入式(1), 并以 u', w', p' 代替 u, w, p , 得到计入惯性项的简化 Navier-Stocks 方程组

$$\begin{cases} \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = - \frac{\partial p'}{\partial x} + \mu \frac{\partial^2 u'}{\partial y^2}; \\ \rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \mu \frac{\partial^2 v'}{\partial y^2}; \\ \rho \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial p'}{\partial z} + \mu \frac{\partial^2 w'}{\partial y^2}. \end{cases} \quad (5)$$

速度边界条件为

$$\begin{cases} y = 0 \text{ 时}, u' = u_1, v' = v_1, w' = w_1 = 0; \\ y = h \text{ 时}, u' = u_2, v' = v_2, w' = w_2 = 0, \end{cases} \quad (6)$$

将式(2), (3), (4)代入式(5), 可求得 u', v', w' . 对直角坐标系下的连续方程, 不计 ρ 随 t 的变化 ($\partial \rho / \partial t = 0$), 沿膜厚进行积分

$$\int_0^h \left(\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} \right) dy = 0, \quad (7)$$

得到含惯性项的雷诺方程, 转为圆柱坐标系形式。

对内膜

$$\begin{aligned} \frac{\partial}{R_1^2 \partial \varphi_1} \left(\frac{h_1^3}{12\mu} \cdot \frac{\partial p'_1}{\partial \varphi_1} \right) + \frac{\partial}{\partial z_1} \left(\frac{h_1^3}{12\mu} \cdot \frac{\partial p'_1}{\partial z_1} \right) = \\ \frac{(\Omega_1 + \Omega_2)}{2} \cdot \frac{\partial h_1}{\partial \varphi_1} + A_1, \end{aligned} \quad (8)$$

有边界条件

$$\begin{cases} y = 0 \text{ 时}, u' = u_1, v' = v_1, w' = 0; \\ y = h \text{ 时}, u' = u_2, v' = v_2, w' = 0. \end{cases}$$

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对外膜

$$\frac{\partial}{\partial \varphi_2} \left(\frac{h_2^3}{12\mu} \cdot \frac{\partial p'_2}{\partial \varphi_2} \right) + \frac{\partial}{\partial z_2} \left(\frac{h_2^3}{12\mu} \cdot \frac{\partial p'_2}{\partial z_2} \right) = \frac{\Omega_2}{2} \cdot \frac{\partial h_2}{\partial \varphi_2} + A_2, \quad (9)$$

有边界条件

$$\begin{cases} y=0 \text{ 时, } u'=u_2, v'=v_2, w'=0; \\ y=h \text{ 时, } u'=0, v'=0, w'=0. \end{cases}$$

2 非定常雷诺方程的无量纲化

2.1 无量纲化因子^[4]

对内膜

$$\bar{\Omega} = \Omega_1 / \Omega_2, \quad \bar{C} = C_2 / C_1, \quad h_1 = H_1 C_1,$$

$$Re_1 = \frac{\rho(\Omega_1 - \Omega_2) R_1 C_1}{\mu}, \quad Re_1^* = \frac{C_1}{R_1} Re_1,$$

$$x_1 = R_1 \varphi_1, \quad \lambda_1 = \frac{z_1}{L_1/2}, \quad P_1 = \frac{\mu(\Omega_1 - \Omega_2) R_1^2}{C_1^2} \bar{P}_1;$$

对外膜

$$Re_2 = \frac{\rho \Omega_2 R_2 C_2}{\mu}, \quad Re_2^* = \frac{C_2}{R_2} Re_2, \quad x_2 = R_2 \varphi_2,$$

$$\lambda_2 = \frac{z_2}{L_2/2}, \quad P_2 = \frac{\mu \Omega_2 R_2^2}{C_2^2} \bar{P}_2.$$

2.2 无量纲化非定常雷诺方程

对内膜

$$\frac{\partial}{\partial \varphi_1} \left(H_1^3 \frac{\partial \bar{P}_1}{\partial \varphi_1} \right) + \left(\frac{D_1^2}{L_1} \right) \frac{\partial}{\partial \lambda_1} \left(H_1^3 \frac{\partial \bar{P}_1}{\partial \lambda_1} \right) = 6 \frac{\bar{\Omega} + 1}{\bar{\Omega} - 1} \cdot \frac{\partial H_1}{\partial \varphi_1} + Re_1^* \bar{A}_1; \quad (10)$$

对外膜

$$\frac{\partial}{\partial \varphi_2} \left(H_2^3 \frac{\partial \bar{P}_2}{\partial \varphi_2} \right) + \left(\frac{D_2^2}{L_2} \right) \frac{\partial}{\partial \lambda_2} \left(H_2^3 \frac{\partial \bar{P}_2}{\partial \lambda_2} \right) = 6 \frac{\partial H_2}{\partial \varphi_2} + Re_2^* \bar{A}_2. \quad (11)$$

$$\begin{aligned} \bar{A}_1 = & \frac{\partial}{\partial \varphi_1} \left\{ -\frac{3H_1^7}{280} \left[\frac{\partial \bar{P}_1}{\partial \varphi_1} \cdot \frac{\partial^2 \bar{P}_1}{\partial \varphi_1^2} + \left(\frac{D_1}{L_1} \right)^2 \right. \right. \\ & \left. \left. \frac{\partial \bar{P}_1}{\partial \lambda_1} \cdot \frac{\partial^2 \bar{P}_1}{\partial \varphi_1 \partial \lambda_1} \right] - \frac{H_1^6}{40} \cdot \frac{\partial H_1}{\partial \varphi_1} \left(\frac{\partial \bar{P}_1}{\partial \varphi_1} \right)^2 + \frac{(9\bar{\Omega} - 4)H_1^4}{20(\bar{\Omega} - 1)} \right. \\ & \left. \frac{\partial \bar{P}_1}{\partial \varphi_1} \cdot \frac{\partial H_1}{\partial \varphi_1} + \frac{(\bar{\Omega} + 1)H_1^5}{20(\bar{\Omega} - 1)} \frac{\partial^2 \bar{P}_1}{\partial \varphi_1^2} - \frac{(8\bar{\Omega} - 3)H_1^2}{10(\bar{\Omega} - 1)} \right. \\ & \left. \frac{\partial H_1}{\partial \varphi_1} - \frac{H_1^4}{40} \left(\frac{D_1}{L_1} \right)^2 \left(H_1^2 \frac{\partial \bar{P}_1}{\partial \varphi_1} - 2 \right) \frac{\partial H_1}{\partial \lambda_1} \cdot \frac{\partial H_1}{\partial \lambda_1} \right\} + \left(\frac{D_1}{L_1} \right)^2 \\ & \frac{\partial}{\partial \lambda_1} \left[-\frac{3H_1^7}{280} \left[\frac{\partial \bar{P}_1}{\partial \varphi_1} \cdot \frac{\partial^2 \bar{P}_1}{\partial \varphi_1 \partial \lambda_1} + \left(\frac{D_1}{L_1} \right)^2 \frac{\partial \bar{P}_1}{\partial \lambda_1} \cdot \frac{\partial^2 \bar{P}_1}{\partial \lambda_1^2} \right] \right. \\ & \left. - \frac{H_1^6}{40} \left(\frac{D_1}{L_1} \right)^2 \left(\frac{\partial \bar{P}_1}{\partial \lambda_1} \right)^2 \frac{\partial H_1}{\partial \lambda_1} - \frac{H_1^6}{40} \cdot \frac{\partial H_1}{\partial \varphi_1} \cdot \frac{\partial \bar{P}_1}{\partial \varphi_1} \cdot \frac{\partial \bar{P}_1}{\partial \lambda_1} + \right. \end{aligned}$$

$$\left. \frac{(\bar{\Omega} + 1)H_1^5}{20(\bar{\Omega} - 1)} \frac{\partial^2 \bar{P}_1}{\partial \varphi_1 \partial \lambda_1} + \frac{(2\bar{\Omega} + 3)H_1^4}{20(\bar{\Omega} - 1)} \cdot \frac{\partial \bar{P}_1}{\partial \lambda_1} \cdot \frac{\partial H_1}{\partial \varphi_1} \right\};$$

$$\bar{A}_2 = \frac{\partial}{\partial \varphi_2} \left\{ -\frac{3H_2^7}{280} \left[\frac{\partial \bar{P}_2}{\partial \varphi_2} \cdot \frac{\partial^2 \bar{P}_2}{\partial \varphi_2^2} + \left(\frac{D_2}{L_2} \right)^2 \frac{\partial \bar{P}_2}{\partial \lambda_2} \right. \right.$$

$$\left. \frac{\partial^2 \bar{P}_2}{\partial \varphi_2 \partial \lambda_2} \right] - \frac{H_2^6}{40} \cdot \frac{\partial H_2}{\partial \varphi_2} \left(\frac{\partial \bar{P}_2}{\partial \varphi_2} \right)^2 + \frac{9H_2^4}{20} \cdot \frac{\partial \bar{P}_2}{\partial \varphi_2} \cdot \frac{\partial H_2}{\partial \varphi_2} +$$

$$\frac{H_2^5}{20} \cdot \frac{\partial^2 \bar{P}_2}{\partial \varphi_2^2} - \frac{4H_2^2}{5} \cdot \frac{\partial H_2}{\partial \varphi_2} - \frac{H_2^4}{40} \cdot \frac{D_2^2}{L_2^2} \left(H_2^2 \frac{\partial \bar{P}_2}{\partial \varphi_2} - 2 \right) \cdot$$

$$\frac{\partial \bar{P}_2}{\partial \lambda_2} \cdot \frac{\partial H_2}{\partial \lambda_2} + \frac{D_2^2}{L_2} \frac{\partial}{\partial \lambda_2} \left\{ -\frac{3H_2^7}{280} \left[\frac{\partial \bar{P}_2}{\partial \varphi_2} \cdot \frac{\partial^2 \bar{P}_2}{\partial \varphi_2 \partial \lambda_2} + \left(\frac{D_2}{L_2} \right)^2 \right. \right.$$

$$\left. \frac{\partial \bar{P}_2}{\partial \lambda_2} \cdot \frac{\partial^2 \bar{P}_2}{\partial \lambda_2^2} \right] - \frac{H_2^6}{40} \left(\frac{D_2}{L_2} \right)^2 \left(\frac{\partial \bar{P}_2}{\partial \lambda_2} \right)^2 \frac{\partial H_2}{\partial \lambda_2} - \frac{H_2^6}{40} \frac{\partial H_2}{\partial \varphi_2}$$

$$\frac{\partial \bar{P}_2}{\partial \varphi_2} \cdot \frac{\partial \bar{P}_2}{\partial \lambda_2} + \frac{H_2^5}{20} \cdot \frac{\partial^2 \bar{P}_2}{\partial \varphi_2 \partial \lambda_2} + \frac{3H_2^4}{20} \cdot \frac{\partial \bar{P}_2}{\partial \lambda_2} \cdot \frac{\partial H_2}{\partial \varphi_2} \left. \right\}.$$

2.3 流量平衡方程及压力边界条件

在此种径向浮环轴承中,外膜为五腔结构,内膜为四腔结构.设轴承和浮环轴端边界为 Γ_1 ,进油边界为 Γ_2 ,深腔边界为 Γ_3 ,则

无量纲压力边界条件

$$\begin{cases} \bar{P}_i = 0 & (i \in \Gamma_1); \\ \bar{P}_j = 1 & (j \in \Gamma_2, \Gamma_3), \end{cases} \quad (12)$$

无量纲流量平衡方程

$$\vec{n} \cdot (6H - H^3 \cdot \nabla \bar{P}) = (1 - \bar{P}_{in}) / \bar{R}_j. \quad (13)$$

3 有限元素方法分析

网格划分采用四边形 8 节点等参单元,取形函数作为权函数,根据迦辽金加权余量法^[3],利用 Green 公式得到余量表达式

$$\begin{aligned} R_a = & \iint_{\Omega^{(e)}} \left[H_i^3 \frac{\partial \bar{P}_i}{\partial \varphi_i} \cdot \frac{\partial W_j}{\partial \varphi_i} + \left(\frac{D_i}{L_i} \right)^2 H_i^3 \frac{\partial \bar{P}_i}{\partial \lambda_i} \cdot \frac{\partial W_j}{\partial \lambda_i} \right] d\Omega - \\ & \iint_{\Omega^{(e)}} \left[6C_i H_i \frac{\partial W_i}{\partial \varphi_i} Re_i^* + \left(R_1 \frac{\partial W_i}{\partial \varphi_i} R_2 \frac{\partial W_i}{\partial \lambda_i} \right) \right] d\Omega. \quad (14) \end{aligned}$$

令 $R_a = 0$, 得到无量纲 Reynolds 有限元方程为

$$[K_i] \cdot [\bar{P}_i] = [F_i] \quad (i = 1, 2). \quad (15)$$

式(15)即为相应的离散线性方程,根据边界条件,可求得压力分布.

4 结果分析

程序采用面向对象编程技术,利用 Visual C++ 6.0 编译,通过对话框输入原始数据,计算结束后形成压力文件,通过 Surfer 绘图软件绘制无量纲压力分布图如图 1、图 2 所示.其上图近似为无惯性压力场,下图为浮环平衡时有惯性压力场,转子转速 $N = 42000 \text{ n/min}$,环速比 $\bar{\Omega} = 0.31$.由图

可见,轴承在高速运转时,流体惯性力直接影响浮环轴承内外膜的壓力分布:①因內膜流体流速高于外膜流体流速,所以流体惯性力对压力分布的影响內膜大于外膜.②流体惯性力使流体压力场

峰值后移,进而会增大浮环轴承静态偏位角.

通过上述分析可知,对于高速或超高速运转的轴承,设计其结构参数时应考虑流体惯性力对轴承结构参数的影响,使轴承结构更加趋于合理.

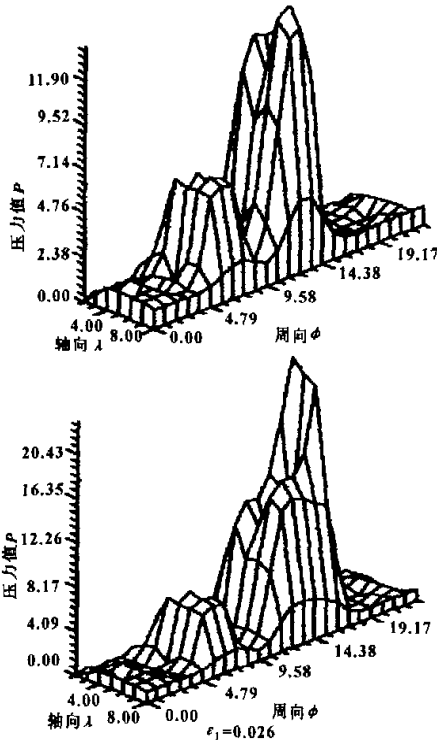


图1 内油膜压力分布

Fig.1 Pressure distribution in inner oil - film

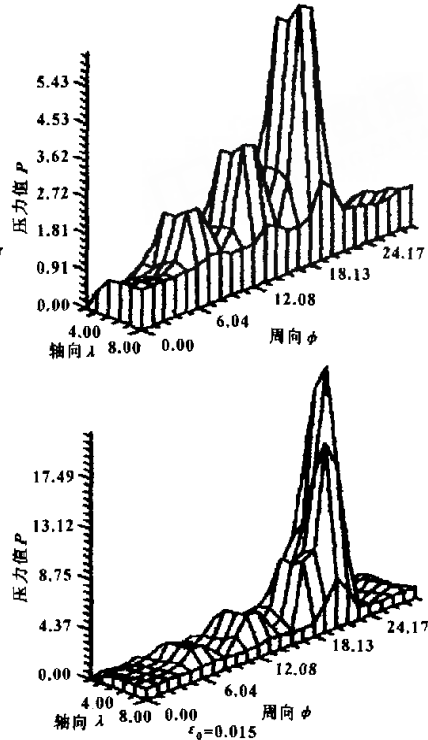


图2 外油膜压力分布

Fig.2 Pressure distribution in outer oil - film

参考文献:

- [1] 张直明.滑动轴承的流体动力润滑理论[M].北京:高等教育出版社,1987.82-83.
- [2] 池长青.流体力学润滑[M].北京:国防工业出版社,

- 1998.407-410.
- [3] 章本照.流体力学中的有限元方法[M].北京:机械工业出版社,1986.229-232.
- [4] 孟凡明.径推浮环轴承静特性研究[D].郑州:郑州工业大学,2000.

The Presure Field of Cylinder Floating Ring Bearing with the Influence of Fluid's Inertiaforce

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Abstract: In this paper, starting from the simplified Navier - Stocks equations, by using of its field of fluid speed and pressure, the non - normal Reynolds equations are given in the form of cylinder coordinates. By means of the finite element method, the pressure field with inertia term is solved. The results show the influence of fluid inertiaforce to this kind of bearing is also more obvious.

Key words: sliding bearing; float - ring; inertia term