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Boundary Element Computation of Stress Intensity Factors for Crack at Bimaterial

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Abstract: Using multi-region Boundary Element Method to get the stress field at the vicinity of a crack, from the relationship between the stress intensity factors K_{I} at bimaterial interface crack tip and the stresses calculated previously, one can get curve $K_{\mathrm{I}}-r$ (r is the polar coordinate), and furthermore, one can get the stress intensity factors from $r=0$. Examples indicates that Boundary Element Method is a valid methed to bimeterial interface crack.

Key words: Boundary Element Method; stress intensity factors; interface cracks

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Introduction

In fact, the bimaterial interface crack can be seen on many engineering structures, such as the crack at the vicinity of gravity dam, the crack at the welded interface between two steel plated, ect. Because of the bimaterial the stress near the crack tip is very complicated[1~5]. As we know the singularity of the stress at the tip of a crack at similar material structure is 1/2, but if the crack is in a different material structure, it is controlled by the properties of the materials, and this make the problem very complicated. By now, finite element method has been used to solve the problem, but the singlar element size is difficult to be determined, and furthermore, the data are much more, which make it not be used widely.

Because of the singularity of fundamental solution, Boundary Element Method is suitable for singular problem, especially for crack problems, the quarter-point boundary element will produce the singularity 1/2, and make the stress intensity factors got easily. However, because the singularity of different material crack is determined by the properties of the two materials, there are a few papers about BEM. In our paper, we use equeal-parameter Boundary Element Method and small element near the tip to imitate the singularity of the bimaterial interface crack, and finally get the stress intensity factors.

1 Essencial Theory

1.1 Relationship between stress intensity factors and stresses

Reference [4] has given the relative formula, if $\theta=0$, the formula are

$$\begin{cases} \sigma_{\pi}^{\alpha} = \dfrac{1}{\sqrt{2\pi r}}\dfrac{3m_\alpha - n_\alpha}{1+R}(K_{\mathrm{I}}\cos t + K_{\mathrm{II}}\sin t); \\ \sigma_{r}^{\alpha} = \dfrac{1}{\sqrt{2\pi r}}(K_{\mathrm{I}}\cos t + K_{\mathrm{II}}\sin t); \\ \sigma_{\pi r}^{\alpha} = \dfrac{1}{\sqrt{2\pi r}}(K_{\mathrm{II}}\cos t - K_{\mathrm{I}}\sin t), \end{cases} \tag{1}$$

where

$$m_\alpha = R^{\alpha-1}, n_\alpha = R^{2-\alpha} \quad (\alpha = 1,2),$$

$$R = \frac{\mu_1 + \mu_2 K_1}{\mu_2 + \mu_1 K_2}, \quad K_\alpha = 3 - 4\nu_\alpha \quad (\text{plane strain}),$$

$$K_\alpha = \frac{3-\nu_\alpha}{1+\nu_\alpha} \quad (\text{plane stress}),$$

$$t = \frac{1}{2\pi}\ln R \cdot \ln r,$$

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here, μ_α, ν_α are material's modul and poisson's ratio respectively.

Polar coordinate system at the cracked tip are shown in Fig.1.

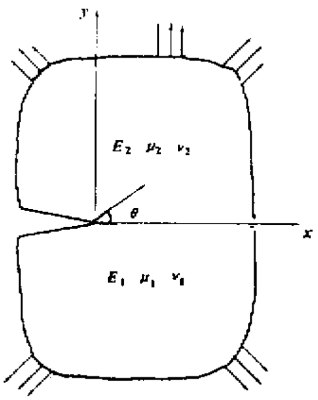

Fig.1　Polar coordinate system at the cracked tip

If $\alpha = 2$, and for plane strain condition, then $m_2 = R$, $n_2 = 1$, $K_2 = 3 - 4v_2$, $K_1 = 3 - 4v_1$. Subtituting m_2, n_2, K_2 and K_1 into Eq.(1), then

$$\sigma_x = \frac{1}{\sqrt{2\pi r}}\frac{3R-1}{1+R}(K_{\mathrm{I}}\cos t + K_{\mathrm{II}}\sin t), \quad (2)$$

$$\sigma_y = \frac{1}{\sqrt{2\pi r}}(K_{\mathrm{I}}\cos t + K_{\mathrm{II}}\sin t), \quad (3)$$

$$\sigma_{xy} = \frac{1}{\sqrt{2\pi r}}(K_{\mathrm{II}}\cos t - K_{\mathrm{I}}\sin t). \quad (4)$$

Suppose the direction of the crack is paralell with coordinate x, then there is no σ_x in this direction, so stress intensity factors can be induced from Eq.(2) and (3), that is

$$\begin{cases} K_{\mathrm{I}} = \sqrt{2\pi r}(\sigma_y\cos t - \sigma_{xy}\sin t); \\ K_{\mathrm{II}} = \sqrt{2\pi r}(\sigma_y\sin t + \sigma_{xy}\cos t). \end{cases} \quad (5)$$

Because non-singular stress is ignored in Eq.(1), Eq.(5)'s suitable range is limited in the area of singular stresses. And therefore stress intensity factors can only be written as

$$\begin{cases} K_{\mathrm{I}} = \lim\limits_{r\to 0} f_{\mathrm{I}}(r); \\ K_{\mathrm{II}} = \lim\limits_{r\to 0} f_{\mathrm{II}}(r), \end{cases} \quad (6)$$

here

$$\begin{cases} f_{\mathrm{I}}(r) = \sqrt{2\pi r}\,(\sigma_y\cos t - \sigma_{xy}\sin t); \\ f_{\mathrm{II}}(r) = \sqrt{2\pi r}\,(\sigma_y\sin t + \sigma_{xy}\cos t). \end{cases} \quad (7)$$

We'll use Eq.(6) to compute the stress intersity factors later.

1.2　Computation of stress intensity factors

Since fundamental solution to BEM is only suitable for similar material, multi-region coupled technique of boundary element method must be used, when a structure is composed of more than one kind of material. Here is the case.

To subregion Ⅰ, BEM Eq. is

$$[H_{\mathrm{I}} \quad H_{\mathrm{I}}^*]\begin{Bmatrix} U_{\mathrm{I}} \\ U_{\mathrm{I}}^* \end{Bmatrix} = [G_{\mathrm{I}} \quad G_{\mathrm{I}}^*]\begin{Bmatrix} T_{\mathrm{I}} \\ T_{\mathrm{I}}^* \end{Bmatrix}, \quad (8)$$

and to subregion Ⅱ, it is

$$[H_{\mathrm{II}} \quad H_{\mathrm{II}}^*]\begin{Bmatrix} U_{\mathrm{II}} \\ U_{\mathrm{II}}^* \end{Bmatrix} = [G_{\mathrm{II}} \quad G_{\mathrm{II}}^*]\begin{Bmatrix} T_{\mathrm{II}} \\ T_{\mathrm{II}}^* \end{Bmatrix}, \quad (9)$$

where, the vector with mark" * "at top right is the corresponding vector at the coupling boundary.

And furthermore, we take the continuous conditions

$$\begin{cases} U_{\mathrm{I}}^* = U_{\mathrm{II}}^* = U, \\ T_{\mathrm{I}}^* = -T_{\mathrm{II}}^* = T, \end{cases} \quad (10)$$

into consideration, Eq.(8), Eq.(9) can be coupled as

$$\begin{bmatrix} H_{\mathrm{I}} & H_{\mathrm{I}}^* & -G_{\mathrm{I}}^* & 0 \\ 0 & H_{\mathrm{II}}^* & G_{\mathrm{II}}^* & H_{\mathrm{II}} \end{bmatrix} = \begin{Bmatrix} U_{\mathrm{I}} \\ U \\ T \\ U_{\mathrm{II}} \end{Bmatrix} =$$

$$\begin{bmatrix} G_{\mathrm{I}} & 0 \\ 0 & G_{\mathrm{II}} \end{bmatrix}\begin{Bmatrix} T_{\mathrm{I}} \\ T_{\mathrm{II}} \end{Bmatrix}. \quad (11)$$

And finally, if one takes the boundary condition into Eq.(11), one can get

$$[A]\{x\} = \{B\}. \quad (12)$$

From Eq.(12), all displacements and tractions of all boundary can be gotten.

Since the singularity of the stresses is not 1/2, the location of the singular point can't be determined previously. In this paper, we use stress external-curved to get stress intensity factors.

To subregion Ⅱ, from the relationship between stress and traction, one can get stress of boundary point, that is

$$\sigma_y = -T_2, \quad \sigma_{xy} = -T_1, \quad (13)$$

here, T_1, T_2 are tractions x and y of coupled boundary point.

Furthermore, stresses of Gaussian point within the first element at the tip of the crack (in our paper, $\xi = -0.95, -0.85, -0.75, -0.65, -0.55$, five points are used). According to Eq.(7), one can get f^j_{I}, f^j_{II}, then curve them as a linear, from $r = 0$, get stress intensity factors:

$$\begin{cases} K_{\mathrm{I}} = (\sum f^j_{\mathrm{I}} \sum f_j^2 - \sum r_j f^j_{\mathrm{I}} \sum r_j) / \\ \quad (5 \sum r_j^2 - \sum r_j \sum r_j) ; \\ K_{\mathrm{II}} = (\sum f^j_{\mathrm{II}} \sum r_j^2 - \sum r_j f^j_{\mathrm{II}} \sum r_j) / \\ \quad (5 \sum r_j^2 - \sum r_j \sum r_j) , \end{cases} \tag{14}$$

where ‘ $\sum$ ’ is a summation from one to five.

2 Numerical Results

2.1 Accuracy

Some examples given in reference [2] have been computed by means of our computer programe, so that the accuracy of this methed can be examined.

A central-cracked plate is shown in Fig.2, loaded by single-direction tensile force, has analytical solution when $w \to \infty$. According to the conclusion come from reference [5], numerical result and analytical solution is resemble, in case of $w/a = 10$. Because of symmetricity, half of the plate is considered.

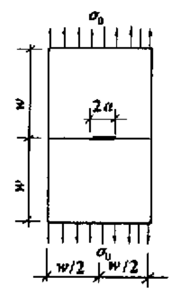

Fig.2 Central – cracked plate

The numerical result is given in table 1, where constant $\nu_1 = \nu_2 = 0.3$, K_0 is complex variable stress intensity factor, that is

$$K = K_{\mathrm{I}} + iK_{\mathrm{II}} , \tag{15}$$

$$K_0 = \sqrt{K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2} , \tag{16}$$

The analytical solution is quoted from reference [2]:

$$\begin{cases} K_{\mathrm{I}}^* = \dfrac{\sigma_0[\cos(\varepsilon \ln 2\alpha) + 2\varepsilon \sin(\varepsilon \ln 2\alpha)]}{\mathrm{coh}\pi\varepsilon} \sqrt{\pi\alpha} ; \\ K_{\mathrm{II}}^* = \dfrac{\sigma_0[\sin(\varepsilon \ln 2\alpha) + 2\varepsilon \cos(\varepsilon \ln 2\alpha)]}{\mathrm{coh}\pi\varepsilon} \sqrt{\pi\alpha} , \end{cases} \tag{17}$$

$$K_0^* = \sqrt{K_{\mathrm{I}}^* + K_{\mathrm{II}}^*} . \tag{18}$$

From table 1, one can see that the relative error between numerical results and analytical solution is within 5%, numerical examples give us the experence that thes method has a good accuracy on condition that two or three small elements which are 1/4 ~ 1/3 of the crack long near the tip of the crack.

2.2 Analysis of central – cracked plate

The size of a practical plate is limited to a certain measurement, in order to analysis these kinds of plates, we have calculated stress intensiry factor of the plate shown in Fig.2, here $w = 10$, $a = 1.0, 2.0, 3.0, 4.0, 5.0$ respectively, constant $\nu_1 = \nu_2 = 0.3$, ratio E_1/E_2 is a varying constant. And the results are listed in table 2. From table 2 one can reach the following conclusion: ①To a same ratio E_1/E_2, stress intensity factor K_0 is increasing as the crack is increasing. ② In the case of the same crack, stress intensity factor K_0 is grently increasing when ratio E_1/E_2 is varying from 1 ~ 2, but later K_0 is grently decreasing and the bigger the ratio E_1/E_2 is, the more gentle K_0 decreases.

Table 1 Relative error of K_0 between numerical and analytical results($\sigma_0 = 1$)

E_1/E_2	K_1	K_2	K_0	K_1^*	K_2^*	K_0^*	error/%
1	1.76308	0.00046	1.76308	1.77250	0.00000	1.77250	-0.53
2	1.75698	0.17988	1.76620	1.76320	0.08578	1.76529	0.05
5	1.69169	0.36919	1.73151	1.73480	0.17011	1.74313	-0.67
10	1.61592	0.51180	1.69503	1.71530	0.20752	1.72781	-1.90
100	1.53741	0.54190	1.63012	1.68960	0.24660	1.70750	-4.53
1000	1.51413	0.57870	1.62095	1.68640	0.25082	1.70495	-4.94

Table 2 SIF K_0 of central-cracked plate ($\sigma_0 = 1$)

E_1/E_2	a				
	1	2	3	4	5
1	1.76308	2.33150	2.88940	3.53552	4.00180
2	1.76620	2.33786	2.87552	3.50192	4.02225
5	1.73151	2.29406	2.79000	3.37815	3.93633
10	1.69503	2.24966	2.71695	3.27455	3.84003
100	1.63012	2.17790	2.59219	3.10816	3.66959
1000	1.62095	2.16808	2.57446	3.08488	3.64488

2.3 SIF of interface crack in gravity dam heel

A heel-cracked gravity dam is shown in Fig.3. The method expressed before is employed to calculate the stress intensity factor K_0 when the length of crack is 1.0 meter, the constants are supposed as follows: $\nu_1 = \nu_2 = 0.167$, $\rho_1 = 0.0$, $\rho_2 = 24000\ \mathrm{N/m^3}$. In which, two conditions are considered, one is that the pressure on the interface of the crack is considered as the linear pressure, the higher eqeals the water pressure, the lower eqeals zero, and the other is the pressure is not considered. Here, the upstream water is surposed to be 80.0 meters high, and results are shown in table 3.

3 Conclusion

BEM is a valid methed for analysis of interface crack in dissimilar structure, which has many advantages than any other method, such as high accuracy, conveneice for using, etc. This method can also be used to solve many other problems, such as ditch crack, dynamic crack, interface crack with trimaterials, and so forth, and these problems are in progress in our researching center.

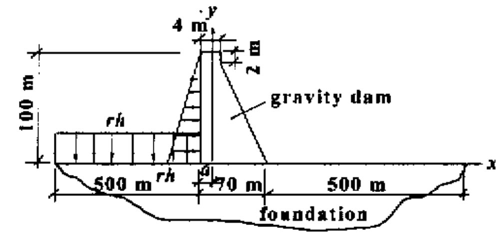

Fig.3 Calculating model of the heel – cracked gravity dam

Table 3 SIF K of interface crack in gravity dam heel

E_1/E_2	no pressure			pressure		
	K_1	K_2	K_0	K_1	K_2	K_0
1.0	–257.32	84.19	270.74	–275.59	147.83	403.64
2.0	–124.36	139.33	186.76	–182.47	231.47	294.74
5.0	–16.51	141.30	142.26	27.11	224.71	226.34
10.0	39.62	122.84	129.07	119.52	188.79	223.44

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边界单元法计算异弹模界面裂缝应力强度因子

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摘　要：利用双区域边界元法计算界面裂缝缝端应力场，用缝端应力强度因子 K_{I} 与应力的关系式求得 $K_{\mathrm{I}}-\gamma$ 曲线（r 为极坐标），从而求得 $r=0$ 处的应力强度因子.算例表明，边界元法是计算界面裂缝的有效方法.

关键词：边界元法；应力强度因子；界面裂缝