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承受 k/r 型剪应力作用楔形体问题的应力解

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摘 要: 对非对称楔形体在楔面承受各种不同荷载作用下的弹性应力解进行了比较系统的研究.首先利用应力函数及其导数在楔形体边界上的力学意义和海维赛(Heaviside)函数,求出了在楔面下部承受 k/r 型剪应力作用下非对称楔形体问题的弹性应力的一般解,然后,令楔面角 α 和 β 取不同的值,可以得到各种不同问题的弹性应力解,如边坡问题和1/4平面问题的弹性应力解.其结果可为实际工程问题提供理论依据.

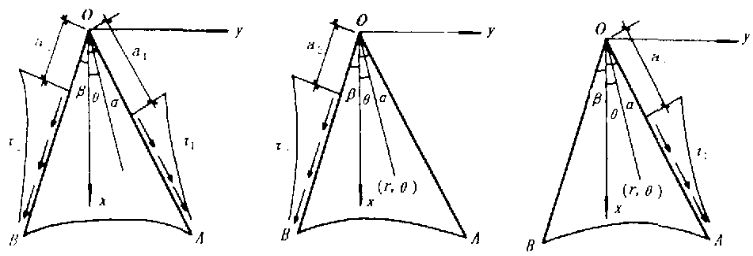
关键词: 楔形体; 剪应力; 应力解

中图分类号: TU 313 **文献标识码**: A

1 力学模型

图1(a)所示为一半无限非对称楔形体,在垂直于纸面方向的厚度为单位1,一楔面 OA 与 x 轴的夹角为 α,另一楔面 OB 与 x 轴的夹角为 β,从楔顶 O,a_1 和 a_2 段长度以下的楔面 OA 和 OB 上分别承受剪应力 $\tau_1=\{r-a_1\}^0k_1/r$ 和 $\tau_2=-\{r-a_2\}^0k_2/r$ 的作用,其中 k_1 和 k_2 均为比例常数.

为计算简便,把图1(a)所示问题看作图1(b)和图1(c)所示两问题的叠加.



(a) $\tau_1=\{r-a_1\}^0\dfrac{k_1}{r}$; $\tau_2=-\{r-a_2\}^0\dfrac{k_2}{r}$ (b) $\tau_2=\{r-a_2\}^0\dfrac{k_2}{r}$ (c) $\tau_1=\{r-a_1\}^0\dfrac{k_1}{r}$

图1 非对称楔形体

2 一般解

先对图1(b)所示问题进行求解,给出弹性应力一般解.然后直接给出图1(c)和图1(a)所示问题的弹性应力一般解.

在图1(b)中,令 A 点为选定的起始点,即

$$(\varphi)_A=0,\left(\frac{\partial\varphi}{\partial\theta}\right)_A=0,$$

则在 AO 边($\theta=\alpha$)上,有

$$\varphi=0,\frac{\partial\varphi}{\partial\theta}=0; \tag{1}$$

在 OB 边($\theta=-\beta$)上,有

$$\varphi=0,$$

$$\frac{1}{r}\frac{\partial\varphi}{\partial\theta}=-Rs=-\{r-a_2\}^0k_2\int_a^r\frac{1}{\xi}\mathrm{d}\xi=-\{r-a_2\}^0k_2(\ln r-\ln a_2), \tag{2}$$

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即

$$\frac{\partial\varphi}{\partial\theta} = -\{r-a_2\}^0 k_2 r\ln r + \{r-a_2\}^0 k_2 r\ln a_2,$$

式中：Rs 为从 A 点到 OB 边界上任一点这段边界上，力在 OB 方向的合力分量[1]；$\{r-a_2\}^0$ 称为梅维赛函数[2]，定义为

$$\{r-a_2\}^0 = \begin{cases} 1, & r \geq a_2; \\ 0, & r < a_2. \end{cases} \tag{3}$$

根据式(2)最后一式的启示，可令图 1(b)所示问题的应力函数 $\varphi(r,\theta)$ 为

$$\varphi(r,\theta) = \varphi_1(r,\theta) + \varphi_2(r,\theta)\ , \tag{4}$$

式中：

$$\varphi_1(r,\theta) = -\{r-a_2\}^0 k_2 r\ln r f_1(\theta)\ ;$$

$$\varphi_2(r,\theta) = \{r-a_2\}^0 k_2 r\ln a_2 f_2(\theta)\ .$$

式中：$f_1(\theta)$，$f_2(\theta)$ 均为 θ 的待定函数，由如下双调和方程

$$\nabla^4\varphi_i(r,\theta) = \nabla^2\nabla^2\varphi_i(r,\theta) = 0 \quad (i=1,2)\ , \tag{5}$$

确定为

$$f_1(\theta) = A_{21}\cos\theta + A_{22}\sin\theta + A_{23}\theta\cos\theta + A_{24}\theta\sin\theta,$$

$$f_2(\theta) = B_{21}\cos\theta + B_{22}\sin\theta + B_{23}\theta\cos\theta + B_{24}\theta\sin\theta.$$

依据上式，可将图 1(b)所示问题的应力函数 $\varphi(r,\theta)$ 写为

$$\varphi(r,\theta) = -\{r-a_2\}^0 k_2[r\ln r(A_{21}\cos\theta + A_{22}\sin\theta + A_{23}\theta\cos\theta + A_{25}\theta\sin\theta) - r\ln a_2(B_{21}\cos\theta + B_{22}\sin\theta + B_{23}\theta\cos\theta + B_{24}\theta\sin\theta)]\ . \tag{6}$$

在以上公式中

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\ ,$$

称为拉普拉斯(laplace)算子. A_{2i}，B_{2i} $(i=1,2,3,4)$ 为待定系数，由式(1)，(2)所示的边界条件确定.

由式(1)、式(2)所示的 4 个边界条件可分别得到 A_{2i} 和 B_{2i} $(i=1,2,3,4)$ 的如下 4 组关系式

$$\begin{cases} A_{21}\cos\alpha + A_{22}\sin\alpha + A_{23}\alpha\cos\alpha + A_{24}\alpha\sin\alpha = 0\ ; \\ B_{21}\cos\alpha + B_{22}\sin\alpha + B_{23}\alpha\cos\alpha + B_{24}\alpha\sin\alpha = 0\ . \end{cases} \tag{7}$$

$$\begin{cases} -A_{21}\sin\alpha + A_{22}\cos\alpha + A_{23}(\cos\alpha - \alpha\sin\alpha) + \\ A_{24}(\sin\alpha + \alpha\cos\alpha) = 0\ ; \\ -B_{21}\sin\alpha + B_{22}\cos\alpha + B_{23}(\cos\alpha - \alpha\sin\alpha) + \\ B_{24}(\sin\alpha + \alpha\cos\alpha) = 0\ . \end{cases} \tag{8}$$

$$\begin{cases} A_{21}\cos\beta - A_{22}\sin\beta - A_{23}\beta\cos\beta + A_{24}\beta\sin\beta = 0\ ; \\ B_{21}\cos\beta - B_{22}\sin\beta - B_{23}\beta\cos\beta + B_{24}\beta\sin\beta = 0\ . \end{cases} \tag{9}$$

$$\begin{cases} A_{21}\sin\beta + A_{22}\cos\beta + A_{23}(\cos\beta - \beta\sin\beta) - \\ A_{24}(\sin\beta + \beta\cos\beta) = 1\ ; \\ B_{21}\sin\beta + B_{22}\cos\beta + B_{23}(\cos\beta - \beta\sin\beta) - \\ B_{24}(\sin\beta + \beta\cos\beta) = 1\ . \end{cases} \tag{10}$$

将上述关系式整理成关于 A_{2i}，B_{2i} 的方程组并进行求解，可得

$$\begin{cases} A_{21} = B_{21} = \dfrac{1}{H}[\alpha\beta\sin\beta - \\ \quad \beta\sin\alpha\cos\alpha\sin\beta + \alpha^2\sin\beta - \beta\sin^2\alpha\cos\beta]\ ; \\ A_{22} = B_{22} = \dfrac{1}{H}[\beta\cos^2\alpha\sin\beta + \\ \quad \alpha\beta\cos\beta + \alpha^2\cos\beta + \beta\sin\alpha\cos\alpha\cos\beta]\ ; \\ A_{23} = B_{23} = \dfrac{-1}{H}[\beta\sin\beta + \alpha\sin\beta + \\ \quad \sin^2\alpha\cos\beta + \sin\alpha\sin\beta\cos\alpha]\ ; \\ A_{24} = B_{24} = \dfrac{1}{H}[\sin\alpha\cos\alpha\cos\beta - \beta\cos\beta - \\ \quad \alpha\cos\beta + \sin\beta\cos^2\alpha]\ ; \\ H = \beta^2 - \cos^2\alpha\sin^2\beta - \sin^2\alpha\cos^2\beta - \\ \quad 2\sin\alpha\sin\beta\cos\alpha\cos\beta + 2\alpha\beta + \alpha^2. \end{cases} \tag{11}$$

上式中，H 为关于 A_{2i}，B_{2i} $(i=1,2,3,4)$ 方程组的系数行列式之值.

根据式(6)，利用应力分量 σ_r，σ_θ，$\tau_{r\theta}$ 与应力函数 $\varphi(r,\theta)$ 间的微分关系，可得图 1(b)所示非对称楔形体问题的弹性应力一般解为

$$\begin{cases} \sigma_r = \dfrac{\partial\varphi}{r\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2\varphi}{\partial\theta^2} = \\ \quad -\{r-a_2\}^0 k_2\{\dfrac{1}{r}[A_{21}\cos\theta + A_{22}\sin\theta + \\ \quad A_{23}\theta\cos\theta + A_{24}\theta\sin\theta] - \\ \quad \dfrac{2(\ln r - \ln a_2)}{r}[A_{23}\sin\theta - A_{24}\cos\theta]\}\ ; \\ \sigma_\theta = \dfrac{\partial^2\varphi}{\partial r^2} = \\ \quad -\{r-a_2\}^0\dfrac{k_2}{r}[A_{21}\cos\theta + A_{22}\sin\theta + \\ \quad A_{23}\theta\cos\theta + A_{24}\theta\sin\theta]\ ; \\ \tau_{r\theta} = -\dfrac{\partial}{\partial r}(\dfrac{1}{r}\dfrac{\partial\varphi}{\partial\theta}) = \\ \quad \{r-a_2\}^0\dfrac{k_2}{r}[A_{21}\sin\theta - A_{22}\cos\theta - \\ \quad A_{23}(\cos\theta - \theta\sin\theta) - A_{24}(\sin\theta + \theta\cos\theta)]\ . \end{cases} \tag{12}$$

用与求解图 1(b)所示问题相同的方法，可求得图 1(c)所示问题的弹性应力一般解为

$$\begin{cases} \sigma_r = \{r - a_1\}^0 \dfrac{k_1}{r}\{[A_{11}\cos\theta + A_{12}\sin\theta + A_{13}\theta\cos\theta + A_{14}\theta\sin\theta] - 2(\ln r - \ln a_1)[A_{13}\sin\theta - A_{14}\cos\theta]\}; \\ \sigma_\theta = \{r - a_1\}^0 \dfrac{k_1}{r}[A_{11}\cos\theta + A_{12}\sin\theta + A_{13}\theta\cos\theta + A_{14}\theta\sin\theta]; \\ \tau_{r\theta} = -\{r - a_1\}^0 \dfrac{k_1}{r}[A_{11}\sin\theta - A_{12}\cos\theta - A_{13}(\cos\theta - \theta\sin\theta) - A_{14}(\sin\theta + \theta\cos\theta)]. \end{cases} \tag{13}$$

式中:

$$\begin{cases} A_{11} = \dfrac{-1}{H}[\alpha\beta\sin\alpha - \alpha\sin\alpha\sin\beta\cos\beta + \beta^2\sin\alpha - \alpha\sin^2\beta\cos\alpha]; \\ A_{12} = \dfrac{1}{H}[\alpha\sin\alpha\cos^2\beta + \alpha\beta\cos\alpha + \beta^2\cos\alpha + \alpha\sin\beta\cos\alpha\cos\beta]; \\ A_{13} = \dfrac{-1}{H}[\alpha\sin\alpha + \beta\sin\alpha + \sin^2\beta\cos\alpha + \sin\alpha\sin\beta\cos\beta]; \\ A_{14} = \dfrac{-1}{H}[\sin\beta\cos\alpha\cos\beta - \alpha\cos\alpha - \beta\cos\alpha + \sin\alpha\cos^2\beta]. \end{cases} \tag{14}$$

显然,图1(a)所示问题的弹性应力一般解为

$$\begin{cases} \sigma_r = \{r - a_1\}^0 \dfrac{k_1}{r}\{[A_{11}\cos\theta + A_{12}\sin\theta + A_{13}\theta\cos\theta + A_{14}\theta\sin\theta] - 2(\ln r - \ln a_1)[A_{13}\sin\theta - A_{14}\cos\theta]\} - \\ \quad \{r - a_2\}^0 \dfrac{k_2}{r}\{[A_{21}\cos\theta + A_{22}\sin\theta + A_{23}\theta\cos\theta + A_{24}\theta\sin\theta] - 2(\ln r - \ln a_2)[A_{23}\sin\theta - A_{24}\cos\theta]\}; \\ \sigma_\theta = \{r - a_1\}^0 \dfrac{k_1}{r}[A_{11}\cos\theta + A_{12}\sin\theta + A_{13}\theta\cos\theta + A_{14}\theta\sin\theta] - \\ \quad \{r - a_2\}^0 \dfrac{k_2}{r}[A_{21}\cos\theta + A_{22}\sin\theta + A_{23}\theta\cos\theta + A_{24}\theta\sin\theta]; \\ \tau_\theta = -\{r - a_1\}^0 \dfrac{k_1}{r}[A_{11}\sin\theta - A_{12}\cos\theta - A_{13}(\cos\theta - \theta\sin\theta) - A_{14}(\sin\theta + \theta\cos\theta)] + \\ \quad \{r - a_2\}^0 \dfrac{k_2}{r}[A_{21}\sin\theta - A_{22}\cos\theta - A_{23}(\cos\theta - \theta\sin\theta) - A_{24}(\sin\theta + \theta\cos\theta)]. \end{cases} \tag{15}$$

3 讨论

式(12)、式(13)和式(15)分别为图1(b)、图1(c)和图1(a)所示非对称楔形体问题的弹性应力一般解,当令 β 或 α 取不同的值时,式(11)、式(14)可得不同的 A_{2i} 和 A_{1i} ($i = 1,2,3,4$),代入式(12)、式(13)、式(15),即得不同楔形体问题的弹性应力解,例如:令 $\beta = \alpha$,可得对称楔形体问题的弹性应力解;令 $\beta = 0$,可得一楔面 OB 铅垂向下的楔形体问题的弹性应力解;令 $\beta = 90°$,可得边坡问题的弹性应力解;令 $\beta = 0°$ 且 $\alpha = 90°$ 或 $\beta = 90°$ 且 $\alpha = 0°$,可得 $\frac{1}{4}$ 平面问题的弹性应力解(应指出的是,相应于 $\beta = 0°$, $\alpha = 90°$ 和 $\beta = 90°$, $\alpha = 0°$ 的两个 $\frac{1}{4}$ 平面问题的弹性应力解,其表达式不同,但存在一定的关系,可参见文献[3~5]).

4 结束语

本文首先利用应力函数及其导数在弹性体边界上的力学意义和海维赛函数的方法,给出了非对称楔面下部承受 k/r 型剪应力作用下问题的弹性应力一般解,然后进行了讨论,并且可以验证,从双调和应力函数 $\varphi(\tau,\theta)$ 出发求得的各应力解满足相应问题的边界条件,所以,各应力分量表示式就是相应问题的弹性应力解.

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Study on Synthesis of 3 – alkylphthalides

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Abstract: 3 – alkylphthalide and its derivatives are isolated as the umbelliferae which are used frequently as ingredints in the prescriptions of traditional chinese medicine. A key medium compound 3 – butyl – 4,7 – dihydrophthalide has been synthesized in two steps starting from cis – Δ^4 – tetrahydrophthalic anhydride, and three kinds of 3 – alkylphthalide have been prepared through different oxydating conditions.

Key words: 3 – butyl – 4,7 – dihydrophthalide; 3 – alkylphtpalide; synthesis

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Stress Solution of the Wedge Under *k/r* Type Shearing Stress

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Abstract: In order to make a systematic study to elasticity stress answer of asymmetric wedge under various loads, and provide a theory basis for engineering problem, The paper utilizes mechanics meaning of stress function and its derivative to the boundary of wedge and the method of Heaviside Fumction, calculaten elasticity stress answer of asymmetric wedge under bear k/r type shearing stress at the bottom of the wedge. And, commanding wedge angle α and β getting various valus, It obtains various problem elasticity stress answers, for example, border slope and 1/4 plane problem elasticity stress answer. The conclusion has both theoretical and practical value.

Key words: wedge; shearing stress; stress general answer