第14卷 第2期 郑州工学院学报 Vol.14 No.2
1993年 6 月 Journal of Zhengzhou Institute of Technology Jun. 1993

任意四边形薄板自然频率的加权残值法计算*

赵 亮 贾生祥

(郑州工学院数理力学系)

摘 要: 本文采用变换的办法解决了加权残值法在计算任意四边形薄板问题时边界条件不易处理的困难，拓广了加权残值法在处理非规则外型固体力学问题方面的应用范围，使工程人员更易掌握板的自然频率的计算.

关键词: 任意四边形板，自然频率，加权残值法.

中图分类号: O34

1 公式推导

样条加权残值法整体解任意四边形薄板的主要困难在于边界条件不易处理. 今采取变换的办法将任意四边形区域化为正方形区域，从而使边界条件便于处理. 下边是本文关于板的公式的具体推导.

设有一任意四边形薄板如图 1 所示

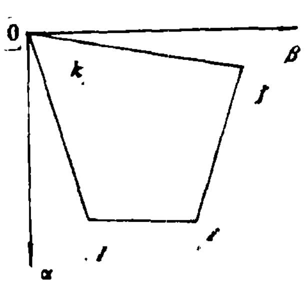

图 1

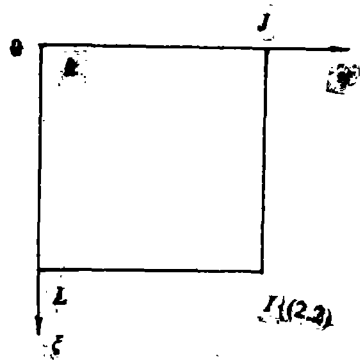

图 2

* 收稿日期: 1992-09-28

取如下变换可将图(1)所示的区域化为图(2)所示的正方形区域:

$$\begin{Bmatrix} \alpha \\ \beta \end{Bmatrix} = \begin{bmatrix} N_i & 0 & N_j & 0 & N_k & 0 & N_L & 0 \\ 0 & N_i & 0 & N_j & 0 & N_k & 0 & N_L \end{bmatrix} \begin{Bmatrix} \alpha_i \\ \beta_i \\ \vdots \\ \alpha_l \\ \beta_L \end{Bmatrix} \tag{1}$$

其中

$$\left.\begin{aligned} N_i &= \frac{1}{4}\zeta\eta \\ N_j &= \frac{1}{4}(2-\zeta)\eta \\ N_k &= \frac{1}{4}(2-\zeta)(2-\eta) \\ N_l &= \frac{1}{4}\zeta(2-\eta) \end{aligned}\right\} \tag{2}$$

设 W(α、β)为薄板的挠度函数，则(1)的变换下有:

$$W(\alpha,\ \beta) = \varphi(\zeta,\eta) \tag{3}$$

$$\begin{aligned} \frac{\partial^4 \omega}{\partial \alpha^4} &= \frac{\partial^4 \varphi}{\partial \zeta^4}\left(\frac{\partial \zeta}{\partial \alpha}\right)^4 + 4\frac{\partial^4 \varphi}{\partial \zeta^3 \partial \eta}\left(\frac{\partial \zeta}{\partial \alpha}\right)^3 \frac{\partial \eta}{\partial \alpha} + 6\frac{\partial^4 \varphi}{\partial \zeta^2 \partial \eta^2}\left(\frac{\partial \zeta}{\partial \alpha}\right)^2\left(\frac{\partial \eta}{\partial \alpha}\right)^2 + 4\frac{\partial^4 \varphi}{\partial \zeta \partial \eta^3}\frac{\partial \zeta}{\partial \alpha}\left(\frac{\partial \eta}{\partial \alpha}\right)^3 \\ &+ 12\frac{\partial^3 \varphi}{\partial \zeta \partial \eta^2}\cdot\frac{\partial \zeta}{\partial \alpha}\cdot\frac{\partial \eta}{\partial \alpha}\cdot\partial^2\eta/\partial\alpha^2 + 6\frac{\partial^3 \varphi}{\partial \eta \partial \zeta^2}\cdot\left(\frac{\partial \zeta}{\partial \alpha}\right)^2\cdot\frac{\partial^2 \eta}{\partial \alpha^2} + 4\frac{\partial^2 \varphi}{\partial \eta \partial \zeta}\cdot\frac{\partial^3 \eta}{\partial \alpha^3}\cdot\frac{\partial \zeta}{\partial \alpha} \\ &+ 6\frac{\partial^3 \varphi}{\partial \eta^3}\cdot\left(\frac{\partial^2 \eta}{\partial \alpha^2}\right)\cdot\left(\frac{\partial \eta}{\partial \alpha}\right)^2 + 4\frac{\partial^2 \varphi}{\partial \eta^2}\cdot\frac{\partial \eta}{\partial \alpha}\cdot\frac{\partial^3 \eta}{\partial \alpha^3} + 3\frac{\partial^2 \varphi}{\partial \eta^2}\cdot\left(\frac{\partial^2 \eta}{\partial \alpha^2}\right)^2 + 4\frac{\partial^4 \varphi}{\partial \eta^4}\cdot\left(\frac{\partial \eta}{\partial \alpha}\right)^4 \\ &+ \frac{\partial \varphi}{\partial \eta}\cdot\frac{\partial^4 \eta}{\partial \alpha^4} \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{\partial^4 W}{\partial \alpha^2 \partial \beta^2} &= \frac{\partial^4 \varphi}{\partial \eta^2 \partial \zeta^2}\cdot\left(\frac{\partial \zeta}{\partial \alpha}\right)^2\cdot\left(\frac{\partial \eta}{\partial \beta}\right)^2 + 2\frac{\partial^4 \varphi}{\partial \eta^3 \partial \zeta}\cdot\frac{\partial \zeta}{\partial \alpha}\cdot\frac{\partial \eta}{\partial \alpha}\cdot\left(\frac{\partial \eta}{\partial \beta}\right)^2 \\ &+ \frac{\partial^4 \varphi}{\partial \eta^4}\cdot\left(\frac{\partial \eta}{\partial \alpha}\right)^2\cdot\left(\frac{\partial \eta}{\partial \beta}\right)^2 + \frac{\partial^3 \varphi}{\partial \eta^3}\cdot\frac{\partial^2 \eta}{\partial \alpha^2}\cdot\left(\frac{\partial \eta}{\partial \beta}\right)^2 + 4\frac{\partial^3 \varphi}{\partial \eta^2 \partial \zeta}\cdot\frac{\partial \zeta}{\partial \alpha}\cdot\frac{\partial \eta}{\partial \beta}\cdot\frac{\partial^2 \eta}{\partial \alpha \partial \beta}\cdot \\ &+ 4\frac{\partial^3 \varphi}{\partial \eta^3}\cdot\frac{\partial \eta}{\partial \alpha}\cdot\frac{\partial \eta}{\partial \beta}\cdot\frac{\partial^2 \eta}{\partial \alpha \partial \beta} + 2\frac{\partial^2 \varphi}{\partial \eta^2}\cdot\left(\frac{\partial^2 \eta}{\partial \alpha \partial \beta}\right)^2 + 2\frac{\partial^2 \varphi}{\partial \eta^2}\cdot\frac{\partial \eta}{\partial \beta}\cdot\frac{\partial^3 \eta}{\partial \beta \partial \alpha^2} \end{aligned} \tag{5}$$

$$\partial^4 W/\partial \beta^4 = \frac{\partial^4 \varphi}{\partial \eta^4}\cdot\frac{\partial^4 \eta}{\partial \beta^4} \tag{6}$$

为了书写上的方便令(4)，(5)，(6)分别为:

$$\frac{\partial^4 W}{\partial \alpha^4} = L_{40}\varphi(\zeta,\ \eta) \tag{7}$$

$$\frac{\partial^4 W}{\partial\alpha^2\partial\beta^2} = L_{22}\varphi(\zeta,\ \eta) \tag{8}$$

$$\partial^4 W / \partial\beta^4 = L_{04}\varphi(\zeta,\ \eta) \tag{9}$$

薄板的自由振动微分方程式为:

$$D\nabla^4 W + \overline{m}\frac{\partial^2 W}{\partial t^2} = 0 \tag{10}$$

式中: $\overline{m}$ 为板中面单位面积的质量，挠度函数 $W = W\ (x,\ y,\ t)$ ，t 为时间．假设薄板在自由振动时系谐振动，则有:

$$W = (A\cos\omega t + B\sin\omega t)\overline{W}(x,y) \tag{11}$$

将(11)代入(10)可得:

$$D\nabla^4 W + \omega^2\overline{m}\overline{W} = 0 \tag{12}$$

将(3)，(7)，(8)和(9)各式代入(12)

$$(L_{40} + 2L_{22} + L_{04})\varphi(\zeta,\eta) - \lambda\varphi(\zeta,\eta) = 0 \tag{13}$$

式(13)就是薄板在图 3 所示的正方形区域上的振动微分方程式．现在只要在图 3 所示的正方形区域上求解式(13)即可，由于是在正方形区域上解式就避开了边界不容易处理的困难．

对式(13)在图 3 所示的结点上进行单样条配点，可得:

$$([G] - \lambda[GO]\ \{a\} = 0 \tag{14}$$

式中: [G]和[GO]是配点得出的系数据阵，由于比较复杂这里就不具体给出了．$\{\alpha\}$是样条试函数的待定系数．

$$\lambda = \omega^2\overline{m} / D \tag{15}$$

方程式(14)要具有非零解{a}，必须有系数矩阵之行列式为零，即:

$$\|[G] - \lambda[GO]\| = 0 \tag{16}$$

从(16)便可解得 λ，从而也就得到了板的自振频率 ω．

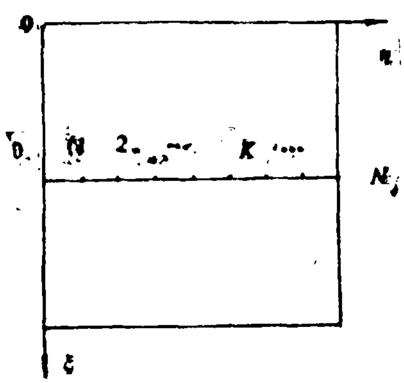

图 3

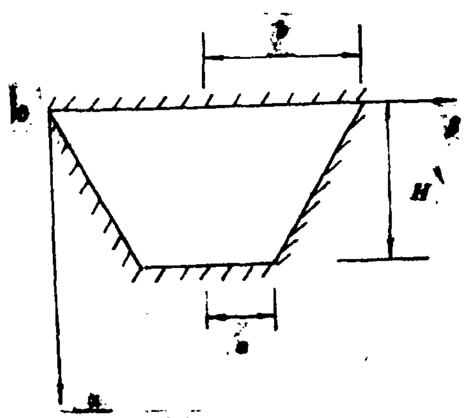

图 4

2 计算实例

已知四边固定的梯形薄板的尺寸为:

$a/b=0.8$ $H/b=12$

如图4所示，求它的最低阶固有频率.

利用能量子域法计算的结果为:

$$\omega=\frac{6.421}{b^2}\sqrt{\frac{D}{\bar{m}}} \tag{17}$$

本文的结果为:

$$\omega=\frac{6.488}{b^2}\sqrt{\frac{D}{\bar{m}}} \tag{18}$$

两者比较相对误差在1%以下.

3 结束语

本文着重在讨论了任意四边形板的动力问题，实际上这种办法亦可推广应用于任意四边形的板壳的稳定问题和弹塑性问题的求解.

通过实际例子的验证表明，本文提出的方法有如下优点:

①精度高，相对误差一般在5%以下.

②速度快，计算时间仅是有限元方法的1/5左右.

③程序简单。

④输入工作量少.

⑤推导过程不涉及复杂的数学工具.

总的来说，本文提出的方法原理简单，易于掌握，适宜于工程实际的应用. 虽然推导过程中有些式子较长，但这并不影响最后应用上的方便.

参考文献

〔1〕 秦荣. 样条最小二乘配点法. 全国加权残值法学术会论文. 1982. 5

〔2〕 林文菁. 徐次达. 样条配点法求结构物动力响应的无条件稳定积分格式及其在网壳结构中的应用. 同济大学力学系. 1985. 3

〔3〕 徐次达主编. 固体力学加权残值法. 同济大学出版社.

〔4〕 秦荣主编. 结构力学的样条函数方法. 广西人民出版社.

〔5〕 Finlayson, B.A. The Method of weighted Residuals and Variational Principles with Application in Fluid mechanics, Heat and Mass Transfer, Acadlmic Press. 1972.

(下转第99页)

Asymptotic behavior of the solution of the neutron trans port equations with continuative energy and with generalized reflecting boundary conditions

Xu Jianguo
(Zhengzhou Institute of Technologg)

Abstract: In this paper, we give the stability theory of the solution of the neutron transport eguations with continuative enevgy and with generalized reflecting boundary conditions. In order to do this, first, we analyze spectrum of transpert operator and prove the existence and the uniqueness of the positive solution of the system given in the Papev. Moreover, we show that the transport operator has at least one real eigenvalue, in fact, which is the dominate eigenvalue. At last, we can indicate the asymptotic behavior of the neutron density as $t\to\infty$ and the asymptotic represent of the neutron distribution in the Hilbert space $L_2(X)$.

Keywords: Traosport equations, Generalized boundary conditions, Transport operator, Dominate eigenvalue, C_0-semigrorp, Asymptotic behavior.

（上接90页）

Calculation of Frequency of Arbitrary Tetragon Plate with Method of Weighted Residuals

Zhao Liang　Jia Sheng Xiang
(Zhengzhou Institute of Thechnology)

Abstract: Dificulty that plate problems in arbitrary tetragon domain were resolved with method of weighted residuals was that boundary conditions were not precessed easily. Arbitrary tetragon domain was changed into square domain with transformation in the essay. The difficulty was overcome. So a new path to calculate integrally frequencies of arbitrary tetragon plate was obtained.

Keywords: Arbitrary Tetragon plate, Frequency, Method of Weighted Residuals