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具有双边丢包和混合时延的 Delta 算子系统 H_∞ 滤波

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摘 要:研究网络控制系统中存在双边丢包和混合随机时延的滤波问题,基于 Delta 算子方法设计了一类 H_∞ 滤波器。假定传感器至控制器、控制器至执行器两个信道存在数据包丢失,由两个独立伯努利分布的白色序列表示。混合随机时延由网络诱导时延和离散无限分布时延组成。利用马尔科夫随机过程构造李雅普诺夫泛函,由线性矩阵不等式得到了具有 H_∞ 性能的滤波误差系统随机稳定的充分条件,给出 H_∞ 滤波器参数。数值算例证明了该方法的有效性。

关键词:网络控制系统;H_∞ 滤波;数据包丢失;混合随机时延;Delta 算子

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0 引言

网络控制系统(networked control system, NCS)由通信技术、网络技术和计算机技术等组成[1]。NCS 具有低耗、安装简便和高可靠性的优点,是研究复杂系统的重要工具[2]。NCS 的应用很多,例如深度神经网络[3]、内视镜检查[4]以及无线传感网络[5]。在数据传输的过程中不可避免会产生时变时延和丢包,给系统带来不利影响,甚至破坏其稳定性[6-7]。与 Kalman 滤波相比,H_∞ 滤波器具有较好的鲁棒性,引起人们的广泛关注[8]。

在控制系统中,时延经常出现会导致系统的性能降低[9]。数据包丢失也会影响系统性能。文献[10-11]讨论了带有数据包丢失的网络控制系统 H_∞ 滤波问题。在鲁棒滤波的早期研究中,基于二次稳定性引入李雅普诺夫方程,以保证该未知系统的鲁棒性能[12]。文献[13]利用伯努利变量描述丢包问题,并建立带丢包的 NCS 模型,设计了 H_∞ 滤波器。文献[14]利用变采样周期的方式,建立网络控制系统,并证明其满足均方指数稳定性。

此外,Middleton 等[15]建立了 Delta 算子方法。Delta 算子具有如下优势:Delta 算子方法将连续与离散系统进行统一处理,在快速采样下 Delta 算子具有更好的数字特性,运用 Delta 算子离散化模型便于观察和分析不同采样周期下的系统性能[16]。

1 问题描述

笔者研究了具有双边丢包和混合随机时延的 Delta 算子描述网络控制系统的 H_∞ 滤波问题。使用马尔科夫链建立滤波误差系统,通过李雅普诺夫泛函证明系统的渐近稳定性和 H_∞ 性能。

Delta 算子定义[15]如下:

$$\delta x(t)=\begin{cases}\dfrac{\mathrm{d}x(t)}{\mathrm{d}t}, & h=0;\\[2mm] \dfrac{x(t+h)-x(t)}{h}, & h\neq 0,\end{cases}$$

式中:h 为采样周期。

考虑下面的 Delta 算子系统:

$$\begin{cases}\delta \boldsymbol{x}(k)=\boldsymbol{A}_\delta \boldsymbol{x}(k)+\boldsymbol{B}_\delta \boldsymbol{w}(k)+\\ \qquad \alpha(k)\boldsymbol{C}_\delta \boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k)))+\\ \qquad \beta(k)\boldsymbol{D}_\delta \sum\limits_{m=1}^{+\infty}\mu_m \boldsymbol{f}(\boldsymbol{x}(k-m));\\ \boldsymbol{y}(k)=\theta(k)\boldsymbol{E}\boldsymbol{x}(k)+\boldsymbol{F}\boldsymbol{w}(k);\\ \boldsymbol{z}(k)=\boldsymbol{L}\boldsymbol{x}(k),\end{cases} \tag{1}$$

式中:$\boldsymbol{x}(k)\in \boldsymbol{R}^n$,表示系统状态向量;$\boldsymbol{\tau}(k)$ 为有界时变时延且是正整数,满足 $\boldsymbol{\tau}_m\leqslant \boldsymbol{\tau}(k)\leqslant \boldsymbol{\tau}_M$,$\boldsymbol{\tau}_m$、$\boldsymbol{\tau}_M$ 分别为时延下界和上界;$\boldsymbol{y}(k)\in \boldsymbol{R}^r$,为系统输出;$\boldsymbol{z}(k)\in \boldsymbol{R}^p$,为要估计的信号;$\boldsymbol{w}(k)\in \boldsymbol{R}^s$,

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为外部扰动，且属于 $L_2[0,\infty)$；$\boldsymbol{g}(\boldsymbol{x}(k))=[\boldsymbol{g}_1(\boldsymbol{x}(k)),\cdots,\boldsymbol{g}_n(\boldsymbol{x}(k))]^{\mathrm{T}}$，$\boldsymbol{f}(\boldsymbol{x}(k))=[\boldsymbol{f}_1(\boldsymbol{x}(k)),\cdots,\boldsymbol{f}_n(\boldsymbol{x}(k))]^{\mathrm{T}}$,为具有适当维数的非线性矩阵。

假设 $\boldsymbol{\tau}(k)$ 取值于有限集合,发生概率如下:

$$\mathrm{Prob}\{\boldsymbol{\tau}_k\}=\lambda_j,j=1,2,\cdots,q,\tag{2}$$

式中:λ_j 是一个正的标量,且 $\sum_{j=1}^{q}\lambda_j=1$。

注 1:网络控制系统由网络将传感器、控制器和执行器连接,形成分散式闭环反馈控制系统,用于远程操控。网络传输可能受到外部非线性因素的影响。

注 2:系统(1)包含无限分布时滞项 $\sum_{m=1}^{+\infty}\mu_m\cdot\boldsymbol{f}(\boldsymbol{x}(k-m))$,它可看成$\int_{-\infty}^{t}k(t-s)\boldsymbol{f}(\boldsymbol{x}(s))\mathrm{d}s$ 的离散化形式[9]。

式(1)中随机变量 $\alpha(k)$、$\beta(k)$、$\theta(k)$ 相互独立,并符合 Bernoulli 分布,满足:

$$\mathrm{Prob}\{\alpha(k)=1\}=\alpha_0;$$
$$\mathrm{Prob}\{\beta(k)=1\}=\beta_0;$$
$$\mathrm{Prob}\{\theta(k)=1\}=\theta_0,$$

其中,$\theta(k)=1$ 代表传感器至控制器的数据传输正常;$\theta(k)=0$ 代表数据包丢失。

考虑如下形式的滤波器:

$$\begin{cases}\delta\boldsymbol{x}_f(k)=\boldsymbol{A}_f\boldsymbol{x}_f(k)+\boldsymbol{B}_f\hat{\boldsymbol{y}}(k);\\ \boldsymbol{z}_f(k)=\boldsymbol{L}_f\boldsymbol{x}_f(k),\end{cases}\tag{3}$$

式中:$\boldsymbol{x}_f(k)\in\boldsymbol{R}^n$ 为滤波器状态;$\hat{\boldsymbol{y}}(k)\in\boldsymbol{R}^q$ 为滤波器输入,$\hat{\boldsymbol{y}}(k)=[\hat{y}_1,\hat{y}_2,\cdots,\hat{y}_q]^{\mathrm{T}}$;$\boldsymbol{z}_f(k)\in\boldsymbol{R}^l$ 为信号 $\boldsymbol{z}(k)$ 的估计;$\boldsymbol{A}_f$、$\boldsymbol{B}_f$、$\boldsymbol{L}_f$ 为待定的滤波器矩阵。

存在通信序列矩阵:

$$\boldsymbol{\Pi}_i=\mathrm{diag}\{\boldsymbol{g}(i-1),\boldsymbol{g}(i-2),\cdots,\boldsymbol{g}(i-q)\},\quad i\in\{1,2,\cdots,q\},\tag{4}$$

式中:$\boldsymbol{g}(k)=\begin{cases}0,k\neq0\\1,k=0\end{cases}$,$\boldsymbol{\Pi}_i$ 表示 k 时刻控制器到执行器进行通信的节点,在 k 时刻成功发送数据,$\hat{\boldsymbol{y}}(k)=\boldsymbol{y}(k)$。

存在数据包丢失时,令$\hat{\boldsymbol{y}}(k)=\boldsymbol{0}$进行处理[8],可得同时含有马尔科夫链和通信序列的滤波器输入:

$$\hat{\boldsymbol{y}}(k)=\boldsymbol{\Pi}_i\boldsymbol{y}(k)。\tag{5}$$

定义向量

$$\boldsymbol{\xi}(k)=[\boldsymbol{x}^{\mathrm{T}}(k)\quad\boldsymbol{x}_f^{\mathrm{T}}(k)];$$
$$\boldsymbol{e}(k)=\boldsymbol{z}(k)-\boldsymbol{z}_f(k)。$$

由式(1)~(5)可以推出滤波误差系统模型:

$$\begin{cases}\delta\boldsymbol{\xi}(k)=(\bar{\boldsymbol{A}}+\tilde{\boldsymbol{A}})\boldsymbol{\xi}(k)+\bar{\boldsymbol{B}}\boldsymbol{w}(k)+\\ \qquad(\bar{\boldsymbol{C}}+\tilde{\boldsymbol{C}})\times\boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k)))+\\ \qquad(\bar{\boldsymbol{D}}+\tilde{\boldsymbol{D}})\times\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m));\\ \boldsymbol{e}(k)=\bar{\boldsymbol{L}}\boldsymbol{\xi}(k),\end{cases}\tag{6}$$

式中:$\bar{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{A}_\delta&\boldsymbol{0}\\\theta_0\boldsymbol{B}_f\boldsymbol{\Pi}_i\boldsymbol{E}&\boldsymbol{A}_f\end{bmatrix}$;$\bar{\boldsymbol{B}}=\begin{bmatrix}\boldsymbol{B}_\delta&\boldsymbol{0}\\\boldsymbol{B}_f\boldsymbol{\Pi}_i\boldsymbol{F}&\boldsymbol{0}\end{bmatrix}$;$\hat{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{0}&\boldsymbol{0}\\\boldsymbol{B}_f\boldsymbol{\Pi}_i\boldsymbol{E}&\boldsymbol{0}\end{bmatrix}$;$\hat{\boldsymbol{C}}=\begin{bmatrix}\boldsymbol{C}_\delta\\\boldsymbol{0}\end{bmatrix}$;$\hat{\boldsymbol{D}}=\begin{bmatrix}\boldsymbol{D}_\delta\\\boldsymbol{0}\end{bmatrix}$;$\bar{\boldsymbol{L}}=[\boldsymbol{L}\quad-\boldsymbol{L}_f]$;$\bar{\boldsymbol{C}}=\alpha_0\hat{\boldsymbol{C}}$;$\bar{\boldsymbol{D}}=\beta_0\hat{\boldsymbol{D}}$;$\tilde{\boldsymbol{A}}=\tilde{\theta}(k)\hat{\boldsymbol{A}}$;$\bar{\boldsymbol{C}}=\tilde{\alpha}(k)\hat{\boldsymbol{C}}$;$\tilde{\boldsymbol{D}}=\tilde{\beta}(k)\hat{\boldsymbol{D}}$;$\tilde{\alpha}(k)=\alpha(k)-\alpha_0$;$\tilde{\beta}(k)=\beta(k)-\beta_0$;$\tilde{\theta}(k)=\theta(k)-\theta_0$。

由此笔者考虑的问题转化成在马尔科夫跳跃系统(6)中滤波器的设计问题。

引理 1[9]　设 $\boldsymbol{M}\in\boldsymbol{R}^{n\times n}$ 是半正定矩阵,$\boldsymbol{x}_i=\boldsymbol{R}^n$,$a_i\geqslant0(i=1,2,\cdots)$。若相关序列是收敛的,则以下不等式成立:

$$\left(\sum_{i=0}^{+\infty}a_i\boldsymbol{x}_i\right)^{\mathrm{T}}\boldsymbol{M}\left(\sum_{i=0}^{+\infty}a_i\boldsymbol{x}_i\right)\leqslant\left(\sum_{i=0}^{+\infty}a_i\right)\sum_{i=0}^{+\infty}a_i\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{M}\boldsymbol{x}_i。\tag{7}$$

现对非常数 $\boldsymbol{g}(\cdot)$ 和 $\boldsymbol{f}(\cdot)$ 进行假设:

假设 1　$\boldsymbol{g}(\cdot)$ 和 $\boldsymbol{f}(\cdot)$ 向量有界,且

$$\boldsymbol{g}(0)=\boldsymbol{f}(0)。$$

假设 2　矩阵 $\boldsymbol{g}(\cdot)$ 和 $\boldsymbol{f}(\cdot)$ 是连续的,且对于 $\forall x,y\in\boldsymbol{R}^n$,有

$$\begin{cases}[\boldsymbol{g}(x)-\boldsymbol{g}(y)-\boldsymbol{Y}_1(x-y)]^{\mathrm{T}}\times\\ \qquad[\boldsymbol{g}(x)-\boldsymbol{g}(y)-\boldsymbol{Y}_2(x-y)]\leqslant0;\\ [\boldsymbol{f}(x)-\boldsymbol{f}(y)-\boldsymbol{V}_1(x-y)]^{\mathrm{T}}\times\\ \qquad[\boldsymbol{f}(x)-\boldsymbol{f}(y)-\boldsymbol{V}_2(x-y)]\leqslant0,\end{cases}\tag{8}$$

式中:$\boldsymbol{Y}_1$、$\boldsymbol{Y}_2$、$\boldsymbol{V}_1$ 和 $\boldsymbol{V}_2$ 为常数矩阵。

2　主要结果

定义 1[8]　当 $\boldsymbol{w}(k)=0$ 时,滤波误差系统是均方渐近稳定的,如果对于任意初始条件,下式成立:

$$\lim_{k\to\infty}E\{\|\boldsymbol{\xi}(k)\|^2\}=0。\tag{9}$$

定理 1　对于离散网络控制系统(1),当 $\boldsymbol{w}(k)=0$ 时,如果存在正定矩阵 $\boldsymbol{P}$,$\boldsymbol{Q}_j$和 $\boldsymbol{Z}_j(j=1,2,\cdots,q)$,使得

$$\boldsymbol{\varphi}=\begin{bmatrix}\boldsymbol{\varphi}_{11}&\boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{Y}}_2&h(h\boldsymbol{A}_\mu^{\mathrm{T}}+\boldsymbol{I})\boldsymbol{\Xi}\boldsymbol{\Gamma}_1&h(h\bar{\boldsymbol{A}}^{\mathrm{T}}+\boldsymbol{I})\overline{\boldsymbol{PD}}\\ *&\boldsymbol{\varphi}_{22}&\boldsymbol{0}&\boldsymbol{0}\\ *&*&\boldsymbol{\varphi}_{33}&h^2\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{D}_\mu\\ *&*&*&\boldsymbol{\varphi}_{44}\end{bmatrix}<0,\tag{10}$$

则滤波误差系统是均方渐近稳定的。式中对称矩

阵中的对称项表示为 * 。

$$\boldsymbol{\varphi}_{11}=h^2(\bar{\boldsymbol{A}}^{\mathrm{T}}+\sigma_\theta\hat{\boldsymbol{A}}^{\mathrm{T}})\boldsymbol{P}(\bar{\boldsymbol{A}}+\sigma_\theta\hat{\boldsymbol{A}})+h((\bar{\boldsymbol{A}}^{\mathrm{T}}+\sigma_\theta\hat{\boldsymbol{A}}^{\mathrm{T}})\boldsymbol{P}+\boldsymbol{P}(\bar{\boldsymbol{A}}+\sigma_\theta\hat{\boldsymbol{A}}))-\boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{Y}}_1\boldsymbol{W}-\boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{V}}_1\boldsymbol{W};$$

$$\boldsymbol{\varphi}_{22}=\sum_{j=1}^{q}(\boldsymbol{Q}_j+\tau_j\boldsymbol{Z}_j);$$

$$\boldsymbol{\varphi}_{33}=h^2(\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{\Gamma}_1+\sigma_\alpha\boldsymbol{\Gamma}_2^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{\Gamma}_2)-\mathrm{diag}\{\boldsymbol{Q}_1,\boldsymbol{Q}_2,\cdots,\boldsymbol{Q}_q\};$$

$$\boldsymbol{\varphi}_{44}=h^2(\bar{\boldsymbol{D}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{D}}+\sigma_\beta\hat{\boldsymbol{D}}^{\mathrm{T}}\boldsymbol{P}\hat{\boldsymbol{D}})-\bar{\mu}^{-1}\boldsymbol{R};$$

$$\bar{\mu}=\sum_{m=1}^{+\infty}\mu_m;$$

$$\boldsymbol{W}=[\boldsymbol{I}\quad \boldsymbol{0}];\boldsymbol{A}_\mu=[\sqrt{\lambda_1}\bar{\boldsymbol{A}}^{\mathrm{T}},\cdots,\sqrt{\lambda_q}\bar{\boldsymbol{A}}^{\mathrm{T}}]^{\mathrm{T}};$$

$$\boldsymbol{B}_\mu=[\sqrt{\lambda_1}\bar{\boldsymbol{B}}^{\mathrm{T}},\cdots,\sqrt{\lambda_q}\bar{\boldsymbol{B}}^{\mathrm{T}}]^{\mathrm{T}};$$

$$\boldsymbol{D}_\mu=[\sqrt{\lambda_1}\bar{\boldsymbol{D}}^{\mathrm{T}},\cdots,\sqrt{\lambda_q}\bar{\boldsymbol{D}}^{\mathrm{T}}]^{\mathrm{T}};$$

$$\boldsymbol{\Gamma}_1=\mathrm{blockdiag}\{\sqrt{\lambda_1}\bar{\boldsymbol{C}}_0,\cdots,\sqrt{\lambda_q}\bar{\boldsymbol{C}}_0\};$$

$$\boldsymbol{\Gamma}_2=\mathrm{blockdiag}\{\sqrt{\lambda_1}\boldsymbol{C}_0,\cdots,\sqrt{\lambda_q}\boldsymbol{C}_0\};$$

$$\boldsymbol{\Xi}=\mathrm{diag}\{\boldsymbol{P},\boldsymbol{P},\cdots\boldsymbol{P}\},\sigma_\alpha=(1-\alpha_0)\alpha_0;$$

$$\sigma_\theta=(1-\theta_0)\theta_0,\sigma_\beta=(1-\beta_0)\beta_0;$$

$$\boldsymbol{C}_0=[\boldsymbol{C}^{\mathrm{T}}\quad \boldsymbol{0}]^{\mathrm{T}},\bar{\boldsymbol{C}}_0=\alpha_0\boldsymbol{C}_0。$$

证明:构造 Delta 域 Lyapunov 泛函:

$$\begin{cases}\boldsymbol{V}(k)=\boldsymbol{V}_1+\boldsymbol{V}_2+\boldsymbol{V}_3+\boldsymbol{V}_4;\\ \boldsymbol{V}_1=\delta\boldsymbol{\xi}^{\mathrm{T}}(k)\boldsymbol{P}\delta\boldsymbol{\xi}(k);\\ \boldsymbol{V}_2=\sum_{j=1}^{q}\sum_{i=k-\tau_j}^{k-1}\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(i))\boldsymbol{Q}_j\boldsymbol{g}(\boldsymbol{x}(i));\\ \boldsymbol{V}_3=\sum_{j=1}^{q}\sum_{i=-\tau_j}^{-1}\sum_{m=k+i}^{k-1}\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(m))\boldsymbol{Z}_j\boldsymbol{g}(\boldsymbol{x}(m));\\ \boldsymbol{V}_4=\sum_{i=1}^{+\infty}\mu_i\sum_{j=k-\tau_j}^{k-1}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(j))\boldsymbol{R}\boldsymbol{f}(\boldsymbol{x}(j))。\end{cases}\qquad(11)$$

当 $\boldsymbol{w}(k)=0$ 时,

$$E\{\Delta\boldsymbol{V}_1\}=E\{\delta\boldsymbol{\xi}^{\mathrm{T}}(k+1)\boldsymbol{P}\delta\boldsymbol{\xi}(k+1)-\delta\boldsymbol{\xi}^{\mathrm{T}}(k)\boldsymbol{P}\delta\boldsymbol{\xi}(k)\}=\boldsymbol{\xi}^{\mathrm{T}}(k)(h^2(\bar{\boldsymbol{A}}^{\mathrm{T}}+\sigma_\theta\hat{\boldsymbol{A}}^{\mathrm{T}})\times\boldsymbol{P}(\bar{\boldsymbol{A}}+\sigma_\theta\hat{\boldsymbol{A}})+h((\bar{\boldsymbol{A}}^{\mathrm{T}}+\sigma_\theta\hat{\boldsymbol{A}}^{\mathrm{T}})\boldsymbol{P}+\boldsymbol{P}(\bar{\boldsymbol{A}}+\sigma_\theta\hat{\boldsymbol{A}})))\boldsymbol{\xi}(k)+\boldsymbol{\xi}^{\mathrm{T}}(k)h(h\boldsymbol{A}_\mu^{\mathrm{T}}+\boldsymbol{I})\times\boldsymbol{\Xi}\boldsymbol{\Gamma}_1\boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k)))+\boldsymbol{\xi}^{\mathrm{T}}(k)h(h\bar{\boldsymbol{A}}^{\mathrm{T}}+\boldsymbol{I})\times\boldsymbol{P}\bar{\boldsymbol{D}}\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m))+h(\bar{\boldsymbol{C}}\boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k))))^{\mathrm{T}}\boldsymbol{P}(h\boldsymbol{A}+\boldsymbol{I})\boldsymbol{\xi}(k)+h\left(\bar{\boldsymbol{D}}\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m))\right)^{\mathrm{T}}\boldsymbol{P}(h\boldsymbol{A}+\boldsymbol{I})\boldsymbol{\xi}(k)+h^2(\boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k))))^{\mathrm{T}}(\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{\Gamma}_1+\sigma_\alpha\boldsymbol{\Gamma}_2^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{\Gamma}_2)\times\boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k)))+h^2(\boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k))))^{\mathrm{T}}\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{D}_\mu\times\left(\bar{\boldsymbol{D}}\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m))\right)^{\mathrm{T}}+h^2((\bar{\boldsymbol{D}}+\sigma_\theta\hat{\boldsymbol{D}})\times\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m)))^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{D}}\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m))+h^2((\bar{\boldsymbol{D}}+\sigma_\theta\hat{\boldsymbol{D}})\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m)))^{\mathrm{T}}\times\boldsymbol{P}\bar{\boldsymbol{C}}\boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k)))。\qquad(12)$$

类似地,

$$E\{\Delta\boldsymbol{V}_2\}=E(\sum_{j=1}^{q}(\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k))\boldsymbol{Q}_j\boldsymbol{g}(\boldsymbol{x}(k))-\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k-\tau_j))\boldsymbol{Q}_j\boldsymbol{g}(\boldsymbol{x}(k-\tau_j))))=\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k))(\sum_{j=1}^{q}\boldsymbol{Q}_j)\boldsymbol{g}(\boldsymbol{x}(k))-\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k-\tau_j))\boldsymbol{Q}_j\boldsymbol{g}(\boldsymbol{x}(k-\tau_j)),\qquad(13)$$

$$E\{\Delta\boldsymbol{V}_3\}=\{E\sum_{j=1}^{q}\sum_{i=-\tau_j}^{-1}(\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k))\boldsymbol{Z}_j\boldsymbol{g}(\boldsymbol{x}(k))-\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k+i))\boldsymbol{Z}_j\boldsymbol{g}(\boldsymbol{x}(k+i)))\}\leqslant\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k))\left(\sum_{j=1}^{q}\tau_j\boldsymbol{Z}_j\right)\boldsymbol{g}(\boldsymbol{x}(k)),\qquad(14)$$

$$E\{\Delta\boldsymbol{V}_4\}\leqslant E\{\bar{\mu}\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(k))\boldsymbol{R}\boldsymbol{f}(\boldsymbol{x}(k))\}-\bar{\mu}^{-1}(\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m)))^{\mathrm{T}}\times\boldsymbol{R}(\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m)))。\qquad(15)$$

由假设1和假设2可直接得到:

$$\begin{cases}-\boldsymbol{\xi}^{\mathrm{T}}(k)\boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{Y}}_1\boldsymbol{W}\boldsymbol{\xi}(k)+2\boldsymbol{\xi}^{\mathrm{T}}(k)\boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{Y}}_2\boldsymbol{f}(\boldsymbol{x}(k))-\boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k))\boldsymbol{g}(\boldsymbol{x}(k))\geqslant0;\\ -\boldsymbol{\xi}^{\mathrm{T}}(k)\boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{V}}_1\boldsymbol{W}\boldsymbol{\xi}(k)+2\boldsymbol{\xi}^{\mathrm{T}}(k)\boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{V}}_2\boldsymbol{f}(\boldsymbol{x}(k))-\boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(k))\boldsymbol{f}(\boldsymbol{x}(k))\geqslant0,\end{cases}\qquad(16)$$

式中:

$\tilde{\boldsymbol{Y}}_1=(\boldsymbol{Y}_1^{\mathrm{T}}\boldsymbol{Y}_2+\boldsymbol{Y}_2^{\mathrm{T}}\boldsymbol{Y}_1)/2;\tilde{\boldsymbol{Y}}_2=(\boldsymbol{Y}_1^{\mathrm{T}}+\boldsymbol{Y}_2^{\mathrm{T}})/2;$

$\tilde{\boldsymbol{V}}_1=(\boldsymbol{V}_1^{\mathrm{T}}\boldsymbol{V}_2+\boldsymbol{V}_2^{\mathrm{T}}\boldsymbol{V}_1)/2;\tilde{\boldsymbol{V}}_2=(\boldsymbol{V}_1^{\mathrm{T}}+\boldsymbol{V}_2^{\mathrm{T}})/2$。

令 $\boldsymbol{\eta}(k)=[\boldsymbol{\xi}^{\mathrm{T}}(k)\quad \boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k))\quad \boldsymbol{g}^{\mathrm{T}}(\boldsymbol{x}(k-\tau_k))\quad \sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m))]^{\mathrm{T}}$,

可以得出:

$$\begin{cases}E\{\Delta\boldsymbol{V}(k)\}\leqslant\boldsymbol{\eta}^{\mathrm{T}}(k)\boldsymbol{\varphi}\boldsymbol{\eta}(k),\\ E\{\Delta\boldsymbol{V}(k)\}\leqslant0。\end{cases}\qquad(17)$$

由 Lyapunov 稳定性理论可知,系统均方稳定。

定义 2[8] 给定标量 $\gamma>0$,滤波误差系统是渐近稳定的且满足 H_∞ 性能 γ,如果在零初始条件下,对于一切非零的 $\boldsymbol{w}(k)\in\boldsymbol{L}_2[0,\infty)$,系统渐近

稳定,并且滤波误差 $\boldsymbol{e}(k)$ 满足:

$$\sum_{k=0}^{+\infty} E\{\|\boldsymbol{e}(k)\|_2\} \leqslant \gamma^2 \sum_{k=0}^{+\infty} E\{\|\boldsymbol{w}(k)\|_2\}。\quad (18)$$

定理 2　给定参数 $\boldsymbol{A}_f,\boldsymbol{B}_f$ 和 $\boldsymbol{L}_f$,设 γ 是一个正常数,且当 $\boldsymbol{w}(k)=0$ 时滤波误差系统是均方稳定的,则在零初始条件下,对于任意非零 $\boldsymbol{w}(k) \in \boldsymbol{L}_2[0,\infty)$,如果存在正定矩阵 $\boldsymbol{P}$、$\boldsymbol{Q}_j$、$\boldsymbol{Z}_j(j=1,2,\cdots,q)$、$\boldsymbol{R}$,使得:

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{Y}}_2 & h(h\boldsymbol{A}_\mu^{\mathrm{T}}+\boldsymbol{I})\boldsymbol{\Xi}\boldsymbol{\Gamma}_1 & h(h\bar{\boldsymbol{A}}^{\mathrm{T}}+\boldsymbol{I})\boldsymbol{P}\bar{\boldsymbol{D}} & h(h(\bar{\boldsymbol{A}}^{\mathrm{T}}+\sigma_\theta\hat{\boldsymbol{A}}^{\mathrm{T}})+\boldsymbol{I})\boldsymbol{P}\bar{\boldsymbol{B}} \\ * & \boldsymbol{\varphi}_{22} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & \boldsymbol{\varphi}_{33} & h^2\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{D}_\mu & h^2\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{B}_\mu \\ * & * & * & \boldsymbol{\varphi}_{44} & h^2\bar{\boldsymbol{D}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{B}} \\ * & * & * & * & \boldsymbol{\varphi}_{55} \end{bmatrix} < 0, \quad (19)$$

则滤波误差系统渐近稳定并具有 H_∞ 性能。式中:$\boldsymbol{\Phi}_{11}=\boldsymbol{\varphi}_{11}+\bar{\boldsymbol{L}}^{\mathrm{T}}\bar{\boldsymbol{L}}$;$\boldsymbol{\varphi}_{55}=h^2\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{B}}-\gamma^2\boldsymbol{I}$;$\boldsymbol{\varphi}_{22}$、$\boldsymbol{\varphi}_{33}$ 和 $\boldsymbol{\varphi}_{44}$ 由式(10)给出。

证明:易证明当 $\boldsymbol{\Phi}<0$ 时 $\boldsymbol{\varphi}<0$,由定理 1 可知,当 $\boldsymbol{w}(k)=0$ 时滤波误差系统是均方渐近稳定的。为研究其在零初始条件下的 H_∞ 性能,由此引入性能指标:

$$J(n)=\sum_{k=0}^{n} E\{\boldsymbol{e}^{\mathrm{T}}(k)\boldsymbol{e}(k)-\gamma^2\boldsymbol{w}^{\mathrm{T}}(k)\boldsymbol{w}(k)\}。\quad (20)$$

构造与定理 1 相同的 Lyapunov 泛函,进行类似的处理,可得

$$\begin{aligned} E\{\Delta\boldsymbol{V}(k)\} \leqslant{} & E\{\boldsymbol{\eta}^{\mathrm{T}}(k)\boldsymbol{\varphi}\boldsymbol{\eta}(k)+h\boldsymbol{w}^{\mathrm{T}}\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}(h(\bar{\boldsymbol{A}}+ \\ & \sigma_\theta\hat{\boldsymbol{A}})+\boldsymbol{I})\boldsymbol{\xi}^{\mathrm{T}}(k)+h^2\boldsymbol{w}^{\mathrm{T}}(k)\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{B}}\boldsymbol{w}(k)+ \\ & h^2\boldsymbol{w}^{\mathrm{T}}(k)\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{C}}_0\boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k)))+h^2\boldsymbol{w}^{\mathrm{T}}(k)\bar{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{P}\times \\ & \bar{\boldsymbol{D}}\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m))+h\boldsymbol{\xi}^{\mathrm{T}}(k)(h(\bar{\boldsymbol{A}}^{\mathrm{T}}+\sigma_\theta\hat{\boldsymbol{A}}^{\mathrm{T}})+\boldsymbol{I})\times \\ & \boldsymbol{P}\bar{\boldsymbol{B}}\boldsymbol{w}(k)+h^2\boldsymbol{g}(\boldsymbol{x}(k-\boldsymbol{\tau}(k)))\boldsymbol{\Gamma}_1^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{B}_\mu\boldsymbol{w}(k)+ \\ & h^2\sum_{m=1}^{+\infty}\mu_m\boldsymbol{f}(\boldsymbol{x}(k-m))\bar{\boldsymbol{D}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{B}}\boldsymbol{w}(k)\}。\quad (21) \end{aligned}$$

则由式(20)~(21)可得

$$\begin{aligned} J(n) ={} & \sum_{k=0}^{n} E\{\boldsymbol{e}^{\mathrm{T}}(k)\boldsymbol{e}(k)-\gamma^2\boldsymbol{w}^{\mathrm{T}}(k)\boldsymbol{w}(k)+\Delta\boldsymbol{V}(k)\} \\ \leqslant{} & \sum_{i=1}^{n} E\{\boldsymbol{\xi}^{\mathrm{T}}(k)\bar{\boldsymbol{L}}^{\mathrm{T}}\bar{\boldsymbol{L}}\boldsymbol{\xi}(k)-\gamma^2\boldsymbol{w}^{\mathrm{T}}(k)\boldsymbol{w}(k)+\Delta\boldsymbol{V}(k)\} \\ ={} & \sum_{k=0}^{n}\bar{\boldsymbol{\eta}}^{\mathrm{T}}(k)\boldsymbol{\Phi}\bar{\boldsymbol{\eta}}(k), \end{aligned}$$

式中:$\bar{\boldsymbol{\eta}}(k)=[\boldsymbol{\eta}^{\mathrm{T}}(k)\quad \boldsymbol{w}^{\mathrm{T}}(k)]^{\mathrm{T}}$。

根据式(19),可推出 $J(n)\leqslant 0$,则 $n\to\infty$ 时可得 $\sum_{k=0}^{+\infty} E\{\|\boldsymbol{e}(k)\|_2\} \leqslant \gamma^2 \sum_{k=0}^{+\infty} E\{\|\boldsymbol{w}(k)\|_2\}$。

定理 3　对于离散网络控制系统,给定常数 $\gamma>0$,如果存在矩阵 $\boldsymbol{P}>0,\boldsymbol{R}>0,\boldsymbol{Q}_j>0,\boldsymbol{Z}_j>0,j=1,2,\cdots,q,\boldsymbol{X}$ 和 $\boldsymbol{L}_f$ 满足:

$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{\Lambda}_1 & \boldsymbol{0} & \boldsymbol{\Lambda}_3 & \boldsymbol{\Lambda}_5 \\ * & \boldsymbol{\Lambda}_2 & \boldsymbol{0} & \boldsymbol{\Lambda}_6 \\ * & * & \boldsymbol{\Lambda}_4 & \boldsymbol{\Lambda}_7 \\ * & * & * & -\boldsymbol{\Xi} \end{bmatrix} < 0, \quad (22)$$

式中:

$$\boldsymbol{\Lambda}_1=\begin{bmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{Y}}_2 \\ * & \boldsymbol{\Lambda}_{22} \end{bmatrix};\boldsymbol{\Lambda}_3=\begin{bmatrix} h\sqrt{\sigma_\theta}\boldsymbol{E}_2^{\mathrm{T}}\boldsymbol{X}^{\mathrm{T}} & \bar{\boldsymbol{L}}^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix};$$

$$\boldsymbol{\Lambda}_5=\begin{bmatrix} h\boldsymbol{\Lambda}_{510} \\ \boldsymbol{0} \end{bmatrix};\boldsymbol{\Lambda}_6=\begin{bmatrix} \boldsymbol{\Xi}\boldsymbol{\Gamma}_1 \\ \boldsymbol{\Xi}\bar{\boldsymbol{D}} \end{bmatrix};\boldsymbol{\Lambda}_7=\begin{bmatrix} h\boldsymbol{\Lambda}_{710} \\ \boldsymbol{0} \end{bmatrix};$$

$$\boldsymbol{\Lambda}_2=\mathrm{diag}\{\boldsymbol{\Lambda}_{33},\boldsymbol{\Lambda}_{44}\};$$

$$\boldsymbol{\Lambda}_4=\mathrm{diag}\{-\gamma^2\boldsymbol{I},-\boldsymbol{P},-\boldsymbol{I}\};$$

$$\begin{aligned}\boldsymbol{\Lambda}_{11}={} & h\sigma_\theta\bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\hat{\boldsymbol{A}}+h\sigma_\theta\hat{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{A}}+h((\bar{\boldsymbol{A}}^{\mathrm{T}}+\sigma_\theta\hat{\boldsymbol{A}}^{\mathrm{T}})\boldsymbol{P}+ \\ & \boldsymbol{P}(\bar{\boldsymbol{A}}+\sigma_\theta\hat{\boldsymbol{A}}))-\boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{Y}}_1\boldsymbol{W}-\boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{V}}_1\boldsymbol{W};\end{aligned}$$

$$\boldsymbol{\Lambda}_{22}=\sum_{j=1}^{q}(\boldsymbol{Q}_j+\tau_j\boldsymbol{Z}_j);$$

$$\boldsymbol{\Lambda}_{33}=h^2\sigma_\alpha\boldsymbol{\Pi}_2^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{\Pi}_2-\mathrm{diag}\{\boldsymbol{Q}_1,\boldsymbol{Q}_2,\cdots,\boldsymbol{Q}_q\};$$

$$\boldsymbol{\Lambda}_{44}=h^2\sigma_\theta\hat{\boldsymbol{D}}^{\mathrm{T}}\boldsymbol{P}\hat{\boldsymbol{D}}-\mu^{-1}\boldsymbol{R};$$

$$\boldsymbol{\Lambda}_{510}=[\sqrt{\lambda_1}\boldsymbol{U}_1,\cdots,\sqrt{\lambda_q}\boldsymbol{U}_1];$$

$$\boldsymbol{\Lambda}_{710}=[\sqrt{\lambda_1}\boldsymbol{U}_2,\cdots,\sqrt{\lambda_q}\boldsymbol{U}_2];$$

$$\boldsymbol{U}_1=\boldsymbol{A}_0{}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{E}_1{}^{\mathrm{T}}\boldsymbol{X}^{\mathrm{T}};$$

$$\boldsymbol{U}_2=\boldsymbol{B}_{10}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{F}_{10}^{\mathrm{T}}\boldsymbol{X}^{\mathrm{T}};\boldsymbol{A}_0=\begin{bmatrix} \boldsymbol{A}_\tau & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix};$$

$$\boldsymbol{E}_1=\begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \theta_0\boldsymbol{E} & \boldsymbol{0} \end{bmatrix};\boldsymbol{E}_2=\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{E} & \boldsymbol{0} \end{bmatrix};\boldsymbol{B}_{10}=\begin{bmatrix} \boldsymbol{B}_\delta \\ \boldsymbol{0} \end{bmatrix};$$

$$\boldsymbol{F}_{10}=\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{F} \end{bmatrix}。$$

若以上条件有可行解,则满足条件的 H_∞ 滤波器参数可由 $\boldsymbol{X}$ 和 $\boldsymbol{L}_f$ 得到:

$$\begin{cases} [\boldsymbol{A}_f \quad \boldsymbol{B}_f]=(\boldsymbol{M}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{M})^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{X}, \\ \bar{\boldsymbol{L}}=[\boldsymbol{L}\quad -\boldsymbol{L}_f]。\end{cases} \quad (23)$$

证明:令

$$\boldsymbol{\phi}_1 = \left[h\boldsymbol{\Xi}\boldsymbol{A}_\mu \quad \boldsymbol{0} \quad \boldsymbol{\Xi}\boldsymbol{\Gamma}_1 \quad \boldsymbol{\Xi}\bar{\boldsymbol{D}} \quad \boldsymbol{\Xi}\bar{\boldsymbol{B}} \right];$$

$$\boldsymbol{\phi}_2 = \left[h\sqrt{\sigma_\theta}\boldsymbol{P}\boldsymbol{B}_f \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \right];$$

$$\boldsymbol{\phi}_3 = \left[\bar{\boldsymbol{L}} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \right],$$

则式(19)等价于:

$$\begin{bmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{W}^{\mathrm{T}}\tilde{\boldsymbol{Y}}_2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & \boldsymbol{\Lambda}_{22} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & \boldsymbol{\Lambda}_{33} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & * & \boldsymbol{\Lambda}_{44} & \boldsymbol{0} \\ * & * & * & * & -\gamma^2\boldsymbol{I} \end{bmatrix} +$$

$$\boldsymbol{\Gamma}_1{}^{\mathrm{T}}\boldsymbol{\Xi}^{-1}\boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_2{}^{\mathrm{T}}\boldsymbol{P}^{-1}\boldsymbol{\Gamma}_2 + \boldsymbol{\Gamma}_3{}^{\mathrm{T}}\boldsymbol{\Gamma}_3 < 0。$$

应用 Schur 补引理[16],得

$$\begin{bmatrix} \boldsymbol{\Lambda}_1 & \boldsymbol{0} & \bar{\boldsymbol{\Lambda}}_3 & \bar{\boldsymbol{\Lambda}}_5 \\ * & \boldsymbol{\Lambda}_2 & \boldsymbol{0} & \boldsymbol{\Lambda}_6 \\ * & * & \boldsymbol{\Lambda}_4 & \bar{\boldsymbol{\Lambda}}_7 \\ * & * & * & -\boldsymbol{\Xi} \end{bmatrix} < 0。$$

其中,$\boldsymbol{\Lambda}_3 = \begin{bmatrix} h\sqrt{\sigma_\theta}\boldsymbol{P}\boldsymbol{B}_f & \bar{\boldsymbol{L}} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$;$\bar{\boldsymbol{\Lambda}}_5 = \begin{bmatrix} h\boldsymbol{\Xi}\boldsymbol{A}_\mu \\ \boldsymbol{0} \end{bmatrix}$;

$\bar{\boldsymbol{\Lambda}}_7 = \begin{bmatrix} \boldsymbol{\Xi}\bar{\boldsymbol{B}} \\ \boldsymbol{0} \end{bmatrix}$。

定理 2 中的参数可以写成如下形式:$\bar{\boldsymbol{A}} = \boldsymbol{A}_0 + \boldsymbol{MKE}_1, \hat{\boldsymbol{B}}_f = \boldsymbol{MKE}_2, \bar{\boldsymbol{B}} = \boldsymbol{B}_{10} + \boldsymbol{MKF}_{10}, \boldsymbol{M} = [\boldsymbol{0} \quad \boldsymbol{I}]^{\mathrm{T}}, \boldsymbol{K} = [\boldsymbol{A}_f \quad \boldsymbol{B}_f], \boldsymbol{X} = \boldsymbol{PMK}$。 证毕。

3 数值仿真

为验证本文方法的可行性,选取如下参数:

$$\boldsymbol{A}_{\delta 1} = \begin{bmatrix} 0.26 & -0.30 \\ 0.50 & -0.60 \end{bmatrix}; \boldsymbol{A}_{\delta 2} = \begin{bmatrix} -0.36 & -0.55 \\ 0.30 & -0.80 \end{bmatrix};$$

$$\boldsymbol{A}_{\delta 3} = \begin{bmatrix} 0.20 & -0.78 \\ 0.89 & 0.30 \end{bmatrix}; \boldsymbol{A}_{\delta 4} = \begin{bmatrix} 0.03 & -0.42 \\ 0.56 & 0.62 \end{bmatrix};$$

$$\boldsymbol{B}_{\delta 1} = \boldsymbol{B}_{\delta 2} = \boldsymbol{B}_{\delta 3} = \boldsymbol{B}_{\delta 4} = \begin{bmatrix} 0.35 & 0.12 \\ 0 & 0 \end{bmatrix};$$

$$\boldsymbol{C}_{\delta 1} = \boldsymbol{C}_{\delta 3} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}; \boldsymbol{C}_{\delta 2} = \boldsymbol{C}_{\delta 4} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix};$$

$$\boldsymbol{D}_{\delta 1} = \begin{bmatrix} 0.2 & 1 \\ 0.2 & 0.3 \end{bmatrix}; \boldsymbol{D}_{\delta 2} = \begin{bmatrix} 0.1 & 0.2 \\ 0. & 0.1 \end{bmatrix};$$

$$\boldsymbol{D}_{\delta 3} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.2 \end{bmatrix}; \boldsymbol{D}_{\delta 4} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix};$$

$$\boldsymbol{E}_1 = \boldsymbol{E}_3 = \boldsymbol{E}_4 = [1 \quad 0], \boldsymbol{E}_2 = [0.6 \quad 0];$$

$$\boldsymbol{F}_1 = [0.5 \quad 1]; \boldsymbol{F}_2 = [0.1 \quad 0.2];$$

$$\boldsymbol{F}_3 = [0 \quad 0.1]; \boldsymbol{F}_4 = [0 \quad 1];$$

$$\boldsymbol{L}_1 = \boldsymbol{L}_2 = \boldsymbol{L}_3 = \boldsymbol{L}_4 = [0.3 \quad 0.8]。$$

情形 1:随机变量 $\alpha(k)$、$\beta(k)$ 和 $\theta(k)$ 分别为

$$\alpha_0 = 0.8; \beta_0 = 0.6; \theta_0 = 0.5。$$

利用 MATLAB 的 LMI 工具箱,通过计算求得 $\gamma = 1.641$,滤波器参数为:

$$\boldsymbol{A}_{f1} = \begin{bmatrix} -0.114 & -0.019 \\ -0.024 & -0.123 \end{bmatrix};$$

$$\boldsymbol{B}_{f1} = \begin{bmatrix} -0.644 & 0.325 \\ 0.421 & -0.213 \end{bmatrix};$$

$$\boldsymbol{L}_{f1} = [0.120 \quad -0.923];$$

$$\boldsymbol{A}_{f2} = \begin{bmatrix} -0.127 & 0.156 \\ 0.151 & -0.074 \end{bmatrix};$$

$$\boldsymbol{B}_{f2} = \begin{bmatrix} -0.661 & -0.227 \\ -0.235 & -0.081 \end{bmatrix};$$

$$\boldsymbol{L}_{f2} = [-0.552 \quad 0.420];$$

$$\boldsymbol{A}_{f3} = \begin{bmatrix} -0.863 & -0.047 \\ -0.033 & -0.791 \end{bmatrix};$$

$$\boldsymbol{B}_{f3} = \begin{bmatrix} 0.099 & -0.406 \\ -0.288 & 0.251 \end{bmatrix};$$

$$\boldsymbol{L}_{f3} = [-0.323 \quad -0.955];$$

$$\boldsymbol{A}_{f4} = \begin{bmatrix} -0.234 & 0.627 \\ 0.545 & 0.357 \end{bmatrix};$$

$$\boldsymbol{B}_{f4} = \begin{bmatrix} -0.766 & -0.771 \\ -0.726 & -0.730 \end{bmatrix};$$

$$\boldsymbol{L}_{f4} = [-0.473 \quad -0.233]。$$

通过计算得到:

$$\sqrt{\sum_{k=0}^{\infty} \|\boldsymbol{e}(k)\|^2 / \|\boldsymbol{w}(k)\|^2} = 1.435 < \gamma。$$

情形 2:随机变量 $\alpha(k)$、$\beta(k)$ 和 $\theta(k)$ 分别为

$$\alpha_0 = 0.5; \beta_0 = 0.7; \theta_0 = 0.5。$$

利用 MATLAB 的 LMI 工具箱,通过计算求得 $\gamma = 1.507$,滤波器参数为:

$$\boldsymbol{A}_{f1} = \begin{bmatrix} -0.593 & -0.022 \\ -0.020 & -0.349 \end{bmatrix};$$

$$\boldsymbol{B}_{f1} = \begin{bmatrix} 0.477 & -0.650 \\ -0.679 & 0.151 \end{bmatrix};$$

$$\boldsymbol{L}_{f1} = [0.815 \quad -0.062];$$

$$\boldsymbol{A}_{f2} = \begin{bmatrix} -0.383 & 0.139 \\ 0.027 & -0.171 \end{bmatrix};$$

$$\boldsymbol{B}_{f2} = \begin{bmatrix} 0.588 & 0.180 \\ -0.352 & -0.311 \end{bmatrix};$$

$$\boldsymbol{L}_{f2} = [0.146 \quad 0.106];$$

$$\boldsymbol{A}_{f3} = \begin{bmatrix} -0.412 & 0.554 \\ 0.502 & 0.834 \end{bmatrix};$$

$$\boldsymbol{B}_{f3}=\begin{bmatrix}0.112 & 0.292\\ 0.063 & -0.187\end{bmatrix};$$

$$\boldsymbol{L}_{f3}=[0.179 \quad -0.536];$$

$$\boldsymbol{A}_{f4}=\begin{bmatrix}-0.479 & -0.017\\ -0.015 & -0.267\end{bmatrix};$$

$$\boldsymbol{B}_{f4}=\begin{bmatrix}-0.140 & 0.073\\ 0.143 & -0.018\end{bmatrix};$$

$$\boldsymbol{L}_{f4}=[-0.164 \quad 0.205]。$$

通过计算得到:

$$\sqrt{\sum_{k=0}^{\infty}\|\boldsymbol{e}(k)\|^2/\|\boldsymbol{w}(k)\|^2}=1.395<\gamma。$$

注 3:由情形 1、情形 2 可知,丢包率的改变在一定程度上影响系统的 H_∞ 性能。当参数取值 $\alpha_0=0,\beta_0=0,\theta_0=1$ 时,可得文献[8]中的结果。

系统的待估信号、滤波器估计信号和滤波误差状态曲线分别如图 1、2 所示。由图可以看出,原系统的待估计信号能很好地被滤波器估计,且因为 $\sqrt{\sum_{k=0}^{\infty}\|\boldsymbol{e}(k)\|^2/\|\boldsymbol{w}(k)\|^2}<\gamma$,证明系统具有 H_∞ 性能,仿真结果可以表明本文方法的可行性。

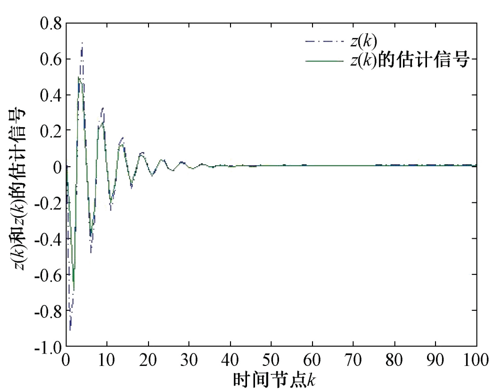

图 1　$z(k)$ 和 $z(k)$ 的估计信号

Figure 1　$z(k)$ and estimated signal of $z(k)$

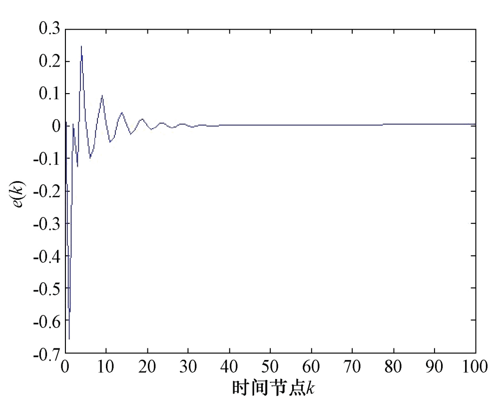

图 2　滤波误差 $\boldsymbol{e}(k)$

Figure 2　Filtering error response $\boldsymbol{e}(k)$

4　结论

研究了具有混合随机时延和双边丢包的 Delta 算子网络控制系统的 H_∞ 滤波问题。通过马尔科夫随机过程,构造基于 Delta 算子的滤波误差系统,采用 Lyapunov 泛函方法,证明该系统的渐近稳定性,并给出滤波器的设计方法。数值算例表明了所提方法的有效性。

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Design of Cam Profile Curve of Flat Bottom Follower Based on ADAMS

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Abstract: When design the contour curve of flat-bottomed follower cam through the method of ADAMS software inversion, the error range was always large which resulted from the unfixed place of contact point between the push rod and the cam, as the generated cam contour was actually the envelope of the push rod contour. To improve the accuracy of the design, the discretization method was proposed for the push rod contour. By taking intersection operation of the multiple curves generated from discrete contact points, ideal cam contour curve was obtained, which could provide a new method for designing flat-bottomed follower cam in ADAMS. Here, by doing the analogue simulation and comparing performance with the design target of the cam mechanism, this paper discussed the relationship between the number of discrete points and the accuracy of design.The results showed that the cam mechanism designed by using the discretization method, the follower had a maximum displacement error of about 0. 207 mm, the follower's thrust was 20 mm, and the maximum error ratio was 1%; the error was mostly within the range of 0. 1 mm, and the overall error ratio was 0. 5%; and the accuracy of cam design could be improved by increasing the number of discrete points.

Key words: flat bottom follower; cam; profile curve; ADAMS;discretization

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H_∞ Filtering for Delta Operator Systems with Two-channel Packet Dropouts and Mixed Delays

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Abstract: In this paper, the problem of H_∞ filtering for networked control systems using delta operator was investigated, which included two-channel packet dropouts and mixed random delays. Random communication packet dropouts existed in channels both from sensors to controllers and from controllers to actuators. They were represented by two independent Bernoulli distributed white sequences. The mixed random time-delays consisted of network induced time delay and discrete infinite distributed delays. A networked-based model was considered with a Markov stochastic process and the H_∞ filtering error system was constructed by using Lyapunov-Krasovskii function in delta domain. A sufficient condition for stochastic stability of the filtering error system with an H_∞ performance was obtained in terms of linear matrix inequalities (LMI). The explicit expression of the desired H_∞ filter was given. A numerical example showed the effectiveness of the proposed method.

Key words: networked control systems; H_∞ filtering; packet dropouts; random time delays; Delta operator