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空心圆柱线圈的电感计算表

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摘 要: 利用轴对称电磁场的性质得到了以矢量磁位为求解对象的边值问题,通过求解该边值问题得到了矢量磁位的表达式,然后利用矢量磁位推导出了空心圆柱线圈电感的计算式,并给出了求解计算式中函数 T 的函数表,以方便精度要求不高时的电感计算,最后用一个实例介绍了函数表的使用,同时验证了利用本文所给函数表求解线圈电感时,计算结果具有更高的精度.

关键词: 空心圆柱线圈; 电感; 矢量磁位; 函数表

中图分类号: TM 153⁺.2 文献标识码: A

0 引言

空心圆柱线圈在电子、电气、无线电等领域中有着广泛的应用.在使用时,常常需要计算线圈的各种相关参数,尤其是线圈的电感.苏联学者 П.Л. 卡兰塔罗夫和 Л.А. 采伊特林给出了空心圆柱线圈的电感计算式及相关的图表^[1].但该式在使用时受到线圈轴向和径向尺寸比值大小的约束,特别是当线圈的长度和线圈的平均直径之比大于 7.2 时,计算精度比较低.因此如何快速、简便、高精度计算空心圆柱线圈的电感就成为一个重要的问题.本文从矢量磁位出发,推导出空心圆柱线圈电感计算的一般性计算式,给出计算式中函数的函数表,从而可以快速、简便地计算空心圆柱线圈的电感.

1 通电空心圆柱线圈的矢量磁位

真空中有一空心圆柱线圈,其内外半径分别为 R_1, R_2 , 轴向长度为 D . 选取圆柱坐标系,原点 o 位于线圈的几何中心, z 轴与线圈的对称轴重合,电流密度 \mathbf{J}_c 的方向和 z 轴正向成右手螺旋关系,线圈的匝流密度为 n_c ,磁导率为 μ_0 ,如图 1 所示.显然该问题满足轴对称场电磁场的定义^[2].为了求出空心圆柱线圈的矢量磁位,先求单匝圆环线圈的矢量磁位.如图 2 所示,设单匝圆环线圈的半径为 ρ' ,线圈所在平面距 $z = 0$ 平面的距离为

z' ,线圈中的电流大小为 I ,方向和 z 轴正向成右手螺旋关系.线圈所在的平面把整个空间区域分成两个场区:场区 1, $z < z'$; 场区 2, $z > z'$.

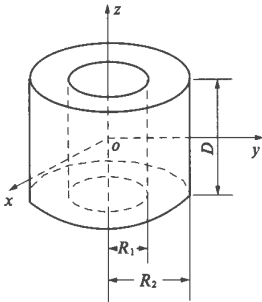


图 1 真空中的空心圆柱线圈
Fig. 1 Air-cored cylindrical coil in vacuum

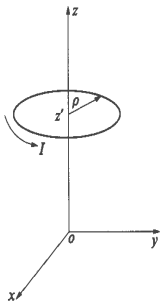


图 2 单匝圆环线圈
Fig. 2 Single-turn coil

由于所求的问题是轴对称电磁场问题,所以

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根据轴对称场的性质知,单匝圆环线圈的矢量磁位 A 只有周向分量 A_ϕ , 其边值问题为^[3] 约束方程:

$$\nabla^2 A_{i\phi} - \frac{1}{\rho^2} A_{i\phi} = 0.$$

式中: $i=1,2$.

边界条件:

$$\lim_{z \rightarrow z_0^-} A_{1\phi} = \lim_{z \rightarrow z_0^+} A_{2\phi};$$
$$\lim_{z \rightarrow z_0^-} \frac{\partial A_{1\phi}}{\partial z} - \lim_{z \rightarrow z_0^+} \frac{\partial A_{2\phi}}{\partial z} = \mu_0 I \delta(\rho - \rho').$$

式中: δ 是狄拉克函数.

无限远条件:

$$\lim_{r \rightarrow \infty} A_{i\phi} = 0.$$

式中: $r = \sqrt{\rho^2 + z^2}$; $i=1,2$.

从而可求出单匝圆环线圈矢量磁位的周向分量为

$$A_{1\phi} = \frac{\mu_0 I \rho'}{2} \int_0^\infty J_1(\lambda \rho') J_1(\lambda \rho) e^{\lambda(z-z')} d\lambda, (z < z')$$
$$A_{2\phi} = \frac{\mu_0 I \rho'}{2} \int_0^\infty J_1(\lambda \rho') J_1(\lambda \rho) e^{-\lambda(z-z')} d\lambda, (z > z'),$$

式中: $J_1(\lambda \rho)$ 是第一类一阶贝塞尔函数.

把空心圆柱线圈看成是无限多个单匝圆环线圈的集合体,定义函数

$$U(R_1, R_2, \lambda) = \frac{1}{\lambda^3} \int_{R_1}^{R_2} t J_1(t) dt \quad (1)$$

利用叠加原理即可得到通电空心圆柱线圈的矢量磁位

$$A = A_\phi \mathbf{e}_\phi = \frac{1}{2} \mu_0 J_c \phi \int_0^\infty U(R_1, R_2, \lambda) J_1(\lambda \rho) [2 - e^{-\lambda(\frac{D}{2}-z)} - e^{-\lambda(z+\frac{D}{2})}] d\lambda \mathbf{e}_\phi \quad (2)$$

2 空心圆柱线圈的电感

通电空心圆柱线圈的矢量磁位仅有周向分量 A_ϕ , 且在任意点 (ρ, ϕ, z) 处的矢量磁位 A 和电流密度 J_c 同方向, 所以^[4]

$$L = \frac{1}{I^2} \int_V \mathbf{A} \cdot \mathbf{J}_c dV'$$
$$= \frac{J_c \phi}{I^2} \int_V \mathbf{A} \cdot \mathbf{e}_\phi dV'$$
$$= \frac{2\pi J_c \phi}{I^2} \int_{R_1}^{R_2} d\rho \int_{-\frac{D}{2}}^{\frac{D}{2}} A_\phi dz \quad (3)$$

把式(2)代入式(3), 并利用式(1)和 $J_c \phi = n_c I$, 可得到

$$L = 2\pi \mu_0 n_c^2 \int_0^\infty U^2(R_1, R_2, \lambda) (\lambda D + e^{-\lambda D} - 1) d\lambda \quad (4)$$

以线圈的内半径 R_1 为基准把线圈的各个长度量归一化, 且令

$$p = \frac{R_2^2}{R_1^2}, q = \frac{D}{R_1}, \lambda = \frac{x}{R_1}.$$

其中, p 和 q 称之为形状参数.

利用以上结果, 式(4)可转化成

$$L = 2\pi \mu_0 n_c^2 R_1^5 T(p, q) \quad (5)$$

式中,

$$T(q, p) = \int_0^\infty U^2(1, p, x) (qx + e^{-qx} - 1) dx \quad (6)$$

式(5)就是所求的空心圆柱线圈的电感计算式.

3 函数 $T(q, p)$ 的计算

利用式(5)求电感时, 最关键的是计算函数 $T(q, p)$. 关于计算函数 $T(q, p)$ 的方法有多种^[4].

如果事先设定一组数

$$q_i = q_0 + h_q i, \quad (i=1, 2, \dots, M);$$

$$p_j = p_0 + h_p j, \quad (j=1, 2, \dots, N).$$

计算对应的函数值 $T(q_i, p_j)$, 就能形成相应的关于 $T(q_i, p_j)$ 的两维函数表, 这里 h_q, h_p 是步长. 表1给出了 $T(q, p)$ 函数表. 利用该函数表能快速、方便地求解空心圆柱线圈的电感. 对于函数表使用作如下说明:

(1) 函数表的最左一列是形状参数 q , 最上一行是形状参数 p .

(2) 函数表中 T 值的最大绝对误差是 5×10^{-5} .

(3) 当所给的形状参数在函数表中没有对应的 T 值时, 使用下面的插值公式计算即可得到所需的 T 值.

$$T(q, p) = T(q_0, p_0) + \frac{T(q_1, p_0) - T(q_0, p_0)}{q_1 - q_0} \cdot$$
$$(q - q_0) + \frac{T(q_0, p_1) - T(q_0, p_0)}{p_1 - p_0} (p - p_0) \quad (7)$$

式中: q 和 p 是待求函数 T 所对应的形状参数, $q_0 < q < q_1, p_0 < p < p_1, q_0$ 和 q_1, p_0 和 p_1 是在函数表中可直接找到的相邻的形状参数.

表 1 $T(q,p)$ 的函数表
Tab.1 Function tables of $T(q,p)$

参数 q	参数 p															
	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75	1.80	
0.10	0.000 1	0.000 1	0.000 2	0.000 3	0.000 5	0.000 6	0.000 8	0.001 0	0.001 2	0.001 4	0.001 8	0.002 1	0.002 4	0.002 7	0.003 2	
0.15	0.000 1	0.000 2	0.000 4	0.000 7	0.001 0	0.001 3	0.001 7	0.002 2	0.002 6	0.003 1	0.003 8	0.004 4	0.005 2	0.005 9	0.006 8	
0.20	0.000 2	0.000 4	0.000 7	0.001 1	0.001 6	0.002 2	0.002 9	0.003 7	0.004 4	0.005 3	0.006 5	0.007 6	0.008 9	0.010 1	0.011 7	
0.25	0.000 3	0.000 6	0.001 0	0.001 7	0.002 5	0.003 4	0.004 3	0.005 5	0.006 6	0.008 0	0.009 8	0.011 5	0.013 5	0.015 4	0.017 7	
0.30	0.000 4	0.000 8	0.001 4	0.002 3	0.003 4	0.004 7	0.006 0	0.007 6	0.009 2	0.011 2	0.013 7	0.016 0	0.018 8	0.021 5	0.024 7	
0.35	0.000 5	0.001 1	0.001 9	0.003 0	0.004 5	0.006 1	0.007 9	0.010 0	0.012 2	0.014 8	0.018 0	0.021 2	0.024 9	0.028 5	0.032 8	
0.40	0.000 6	0.001 3	0.002 4	0.003 8	0.005 6	0.007 7	0.010 0	0.012 7	0.015 5	0.018 7	0.022 9	0.026 9	0.031 6	0.036 2	0.041 7	
0.45	0.000 7	0.001 6	0.002 9	0.004 7	0.006 9	0.009 4	0.012 2	0.015 6	0.019 0	0.023 1	0.028 2	0.033 2	0.039 0	0.044 7	0.051 4	
0.50	0.000 9	0.001 9	0.003 5	0.005 6	0.008 2	0.011 3	0.014 7	0.018 7	0.022 9	0.027 8	0.033 9	0.039 9	0.046 9	0.053 9	0.062 0	
0.55	0.001 0	0.002 3	0.004 1	0.006 6	0.009 7	0.013 3	0.01 72	0.022 0	0.026 9	0.032 8	0.040 0	0.047 1	0.055 4	0.063 7	0.073 3	
0.60	0.001 2	0.002 6	0.004 7	0.007 6	0.011 2	0.015 3	0.020 0	0.025 5	0.031 3	0.038 0	0.046 4	0.054 8	0.064 4	0.074 1	0.085 3	
0.65	0.001 3	0.003 0	0.005 4	0.008 7	0.012 7	0.017 5	0.022 8	0.029 2	0.035 8	0.043 6	0.053 2	0.062 8	0.073 9	0.085 0	0.097 9	
0.70	0.001 5	0.003 4	0.006 1	0.009 8	0.014 4	0.019 8	0.025 8	0.033 0	0.040 5	0.049 4	0.060 2	0.071 2	0.083 8	0.096 5	0.111 1	
0.75	0.001 7	0.003 8	0.006 8	0.010 9	0.016 1	0.022 1	0.028 9	0.037 0	0.045 5	0.055 5	0.067 6	0.080 0	0.094 1	0.108 5	0.125 0	
0.80	0.001 9	0.004 2	0.007 5	0.012 1	0.017 8	0.024 6	0.032 1	0.041 1	0.050 6	0.061 7	0.075 3	0.08 91	0.104 9	0.120 9	0.139 4	
0.85	0.002 0	0.004 6	0.008 3	0.013 3	0.019 6	0.027 1	0.035 4	0.045 3	0.055 9	0.068 2	0.083 2	0.098 5	0.116 0	0.133 8	0.154 2	
0.90	0.002 2	0.005 0	0.009 1	0.014 6	0.021 5	0.029 6	0.038 8	0.049 7	0.061 3	0.074 9	0.091 3	0.108 2	0.127 4	0.147 1	0.169 6	
0.95	0.002 4	0.005 4	0.009 9	0.015 9	0.023 4	0.032 2	0.042 2	0.054 1	0.066 9	0.081 7	0.099 7	0.118 2	0.139 2	0.160 8	0.185 5	
1.00	0.002 6	0.005 9	0.010 7	0.017 2	0.025 3	0.034 9	0.045 8	0.058 7	0.072 6	0.088 8	0.108 2	0.128 4	0.151 3	0.174 9	0.201 7	
1.05	0.002 8	0.006 3	0.011 5	0.018 5	0.027 3	0.037 7	0.049 4	0.063 4	0.078 4	0.096 0	0.117 0	0.138 9	0.163 7	0.189 3	0.218 4	
1.10	0.003 0	0.006 8	0.012 3	0.019 9	0.029 3	0.040 5	0.053 1	0.068 2	0.084 4	0.103 3	0.126 0	0.149 6	0.176 4	0.204 0	0.235 5	
1.15	0.003 2	0.007 3	0.013 2	0.021 3	0.031 3	0.043 3	0.056 9	0.073 0	0.090 5	0.110 8	0.135 1	0.160 5	0.189 3	0.219 1	0.252 9	
1.20	0.003 4	0.007 7	0.014 1	0.022 7	0.033 4	0.046 2	0.060 7	0.078 0	0.096 7	0.118 4	0.144 5	0.171 6	0.202 5	0.234 5	0.270 7	
1.25	0.003 6	0.008 2	0.014 9	0.024 1	0.035 5	0.049 1	0.064 6	0.083 0	0.102 9	0.126 2	0.153 9	0.183 0	0.215 9	0.250 1	0.288 8	
1.30	0.003 9	0.008 7	0.015 8	0.025 5	0.037 6	0.052 1	0.068 5	0.088 1	0.109 3	0.134 1	0.163 6	0.194 5	0.229 5	0.266 0	0.307 3	
1.35	0.004 1	0.009 2	0.016 7	0.027 0	0.039 8	0.055 1	0.072 5	0.093 3	0.115 8	0.142 1	0.173 3	0.206 2	0.243 4	0.282 1	0.326 0	
1.40	0.004 3	0.009 7	0.017 7	0.028 5	0.042 0	0.058 2	0.076 6	0.098 5	0.122 4	0.150 2	0.183 2	0.218 0	0.257 4	0.298 5	0.345 0	
1.45	0.004 5	0.010 2	0.018 6	0.030 0	0.044 2	0.061 2	0.080 7	0.103 8	0.129 0	0.158 4	0.193 3	0.230 0	0.271 7	0.315 2	0.364 3	
1.50	0.004 7	0.010 7	0.019 5	0.031 5	0.046 4	0.064 4	0.084 8	0.109 2	0.135 7	0.166 7	0.203 4	0.242 2	0.286 1	0.332 0	0.383 9	
1.55	0.005 0	0.011 2	0.020 4	0.033 0	0.048 7	0.067 5	0.089 0	0.114 6	0.142 5	0.175 1	0.213 7	0.254 5	0.300 7	0.349 1	0.403 7	
1.60	0.005 2	0.011 7	0.021 4	0.034 5	0.051 0	0.070 7	0.093 2	0.120 1	0.149 4	0.183 5	0.224 1	0.267 0	0.315 5	0.366 3	0.423 7	
1.65	0.005 4	0.012 3	0.022 3	0.036 1	0.053 3	0.073 9	0.097 5	0.125 6	0.156 3	0.192 1	0.234 6	0.279 5	0.330 4	0.383 7	0.44 40	
1.70	0.005 6	0.012 8	0.023 3	0.037 6	0.055 6	0.077 1	0.101 8	0.131 2	0.163 3	0.200 7	0.245 2	0.292 2	0.345 4	0.401 3	0.464 4	
1.75	0.005 9	0.013 3	0.024 3	0.039 2	0.057 9	0.080 4	0.106 1	0.136 8	0.170 3	0.209 5	0.255 8	0.305 0	0.360 6	0.419 1	0.485 1	
1.80	0.006 1	0.013 8	0.025 2	0.040 8	0.060 3	0.083 6	0.110 5	0.142 4	0.177 4	0.218 3	0.266 6	0.317 9	0.376 0	0.437 1	0.506 0	
1.85	0.006 3	0.014 4	0.026 2	0.042 4	0.062 6	0.086 9	0.114 9	0.148 1	0.184 6	0.227 1	0.277 5	0.331 0	0.391 5	0.455 2	0.527 0	
1.90	0.006 6	0.014 9	0.027 2	0.044 0	0.065 0	0.090 2	0.119 3	0.153 8	0.191 8	0.236 0	0.288 4	0.344 1	0.407 1	0.473 4	0.548 2	
1.95	0.006 8	0.015 4	0.028 2	0.045 6	0.067 4	0.093 6	0.123 7	0.159 6	0.199 1	0.245 0	0.299 4	0.357 3	0.422 8	0.491 8	0.569 6	
2.00	0.007 0	0.016 0	0.029 2	0.047 2	0.069 8	0.096 9	0.128 2	0.165 4	0.206 4	0.254 1	0.310 5	0.370 6	0.438 6	0.510 3	0.591 2	
2.05	0.007 3	0.016 5	0.030 2	0.048 8	0.072 2	0.100 3	0.132 7	0.171 3	0.213 7	0.263 2	0.321 7	0.384 0	0.454 5	0.529 0	0.612 9	
2.10	0.007 5	0.017 1	0.031 2	0.050 4	0.074 6	0.103 7	0.137 2	0.177 1	0.221 1	0.272 3	0.332 9	0.397 5	0.470 6	0.547 8	0.634 7	
2.15	0.007 7	0.017 6	0.032 2	0.052 1	0.077 1	0.107 1	0.141 8	0.183 0	0.228 5	0.281 5	0.344 2	0.411 1	0.486 7	0.566 7	0.656 7	
2.20	0.008 0	0.018 2	0.033 2	0.053 7	0.079 5	0.110 5	0.146 3	0.189 0	0.236 0	0.290 8	0.355 6	0.424 7	0.502 9	0.585 7	0.678 9	
2.25	0.008 2	0.018 7	0.034 2	0.055 4	0.082 0	0.114 0	0.150 9	0.194 9	0.243 5	0.300 1	0.367 0	0.438 4	0.519 2	0.604 8	0.701 1	
2.30	0.008 5	0.019 3	0.035 3	0.057 0	0.084 4	0.117 4	0.155 5	0.200 9	0.251 1	0.309 5	0.378 5	0.452 2	0.535 6	0.624 0	0.723 5	
2.35	0.008 7	0.019 8	0.036 3	0.058 7	0.086 9	0.120 9	0.160 2	0.206 9	0.258 6	0.318 8	0.390 0	0.466 1	0.552 1	0.643 3	0.7460	
2.40	0.009 0	0.020 4	0.037 3	0.060 4	0.089 4	0.124 4	0.164 8	0.213 0	0.266 2	0.328 3	0.401 6	0.480 0	0.568 7	0.662 8	0.768 7	
2.45	0.009 2	0.021 0	0.038 4	0.062 0	0.091 9	0.127 9	0.169 5	0.219 0	0.273 9	0.337 8	0.413 2	0.494 0	0.585 3	0.682 3	0.791 4	
2.50	0.009 4	0.021 5	0.039 4	0.063 7	0.094 4	0.131 4	0.174 1	0.225 1	0.281 5	0.347 3	0.424 9	0.508 0	0.602 0	0.701 9	0.814 3	
2.55	0.0097	0.0221	0.0404	0.0654	0.0969	0.1349	0.1788	0.2312	0.289 2	0.356 8	0.436 6	0.522 1	0.618 8	0.721 6	0.837 2	
2.60	0.009 9	0.022 7	0.041 5	0.067 1	0.099 4	0.138 4	0.183 6	0.237 3	0.296 9	0.366 4	0.448 4	0.536 3	0.635 7	0.741 3	0.860 2	
2.65	0.010 2	0.023 2	0.042 5	0.068 8	0.101 9	0.142 0	0.188 3	0.243 5	0.304 7	0.376 0	0.460 2	0.550 5	0.652 6	0.761 2	0.883 4	
2.70	0.010 4	0.023 8	0.043 6	0.070 5	0.104 5	0.145 5	0.193 0	0.249 7	0.312 5	0.385 7	0.472 0	0.564 7	0.669 6	0.781 1	0.906 6	
2.75	0.010 7	0.024 4	0.044 6	0.072 2	0.107 0	0.14 91	0.197 8	0.255 8	0.320 3	0.395 3	0.483 9	0.579 0	0.686 6	0.801 1	0.929 9	
2.80	0.010 9	0.024 9	0.045 7	0.073 9	0.109 5	0.152 6	0.202 5	0.262 0	0.328 1	0.405 0	0.495 8	0.593 4	0.703 7	0.821 1	0.953 3	
2.85	0.011 2	0.025 5	0.046 7	0.075 6	0.112 1	0.156 2	0.207 3	0.268 3	0.335 9	0.414 8	0.507 8	0.607 8	0.720 9	0.841 3	0.976 8	
2.90	0.011 4	0.026 1	0.047 8	0.077 3	0.114 7	0.159 8	0.212 1	0.274 5	0.343 8	0.424 5	0.519 8	0.622 2	0.738 1	0.861 5	1.000 4	
2.95	0.011 7	0.026 6	0.048 8	0.079 0	0.117 2	0.163 4	0.216 9	0.280 7	0.351 7	0.434 3	0.531 8	0.636 7	0.755 3	0.881 7	1.024 0	
3.00	0.011 9	0.027 2	0.049 9	0.080 8	0.119 8	0.167 0	0.221 7	0.287 0	0.359 6	0.444 1	0.543 9	0.651 2	0.772 6	0.902 0	1.047 7	
3.05	0.012 2	0.027 8	0.050 9	0.082 5	0.122 4	0.170 6	0.226 5	0.293 3	0.367 5	0.454 0	0.556 0	0.665 8	0.790 0	0.922 4	1.071 5	

续表

参数 q	参数 p														
	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75	1.80
3.10	0.012 4	0.028 4	0.052 0	0.084 2	0.124 9	0.174 2	0.231 4	0.299 6	0.375 4	0.463 8	0.568 1	0.680 4	0.807 4	0.942 8	1.095 3
3.15	0.012 7	0.029 0	0.053 1	0.086 0	0.127 5	0.177 8	0.236 2	0.305 9	0.383 4	0.473 7	0.580 2	0.695 0	0.824 8	0.963 3	1.119 2
3.20	0.012 9	0.029 5	0.054 1	0.087 7	0.130 1	0.181 4	0.241 1	0.312 2	0.391 3	0.483 6	0.592 4	0.709 7	0.842 3	0.983 9	1.143 2
3.25	0.013 2	0.030 1	0.055 2	0.089 4	0.132 7	0.185 1	0.245 9	0.318 5	0.399 3	0.493 6	0.604 6	0.724 4	0.859 8	1.004 4	1.167 2
3.30	0.013 4	0.030 7	0.056 3	0.091 2	0.135 3	0.188 7	0.250 8	0.324 8	0.407 3	0.503 5	0.616 9	0.739 1	0.877 4	1.025 1	1.191 3
3.35	0.013 7	0.031 3	0.057 4	0.092 9	0.137 9	0.192 3	0.255 7	0.331 2	0.415 4	0.513 5	0.629 1	0.753 8	0.895 0	1.045 8	1.2154
3.40	0.013 9	0.031 9	0.058 4	0.094 7	0.140 5	0.196 0	0.260 5	0.337 6	0.423 4	0.523 5	0.641 4	0.768 6	0.912 6	1.066 5	1.239 6
3.45	0.014 2	0.032 4	0.059 5	0.096 4	0.143 1	0.199 6	0.265 4	0.343 9	0.431 4	0.533 5	0.653 7	0.783 5	0.930 3	1.087 3	1.263 9
3.50	0.014 4	0.033 0	0.060 6	0.098 2	0.145 7	0.203 3	0.270 3	0.350 3	0.439 5	0.543 5	0.666 0	0.798 3	0.948 0	1.108 1	1.288 2
3.55	0.014 7	0.033 6	0.061 7	0.099 9	0.148 3	0.207 0	0.275 2	0.356 7	0.447 6	0.553 5	0.678 4	0.813 2	0.965 7	1.128 9	1.312 5
3.60	0.014 9	0.034 2	0.062 7	0.101 7	0.150 9	0.210 6	0.280 2	0.363 1	0.455 7	0.563 6	0.690 7	0.828 1	0.983 5	1.149 8	1.336 9
3.65	0.015 2	0.034 8	0.063 8	0.103 4	0.153 6	0.214 3	0.285 1	0.369 5	0.463 8	0.573 7	0.703 1	0.843 0	1.001 3	1.170 7	1.361 4
3.70	0.015 4	0.035 4	0.064 9	0.105 2	0.156 2	0.218 0	0.290 0	0.375 9	0.471 9	0.583 7	0.715 5	0.858 0	1.019 2	1.191 7	1.385 9
3.75	0.015 7	0.036 0	0.066 0	0.106 9	0.158 8	0.221 7	0.294 9	0.382 4	0.480 0	0.593 8	0.727 9	0.872 9	1.037 0	1.212 7	1.410 4
3.80	0.016 0	0.036 5	0.067 1	0.108 7	0.161 4	0.225 4	0.299 9	0.388 8	0.488 1	0.604 0	0.740 4	0.887 9	1.054 9	1.2338	1.435 0
3.85	0.016 2	0.037 1	0.068 1	0.110 5	0.164 1	0.229 1	0.304 8	0.395 2	0.496 3	0.614 1	0.752 9	0.903 0	1.072 8	1.254 8	1.459 6
3.90	0.016 5	0.037 7	0.069 2	0.112 2	0.166 7	0.232 8	0.309 8	0.401 7	0.504 4	0.624 2	0.765 3	0.918 0	1.090 8	1.275 9	1.484 3
3.95	0.016 7	0.038 3	0.070 3	0.114 0	0.169 3	0.236 5	0.314 7	0.408 2	0.512 6	0.634 4	0.777 8	0.933 1	1.108 8	1.297 1	1.509 0
4.00	0.017 0	0.038 9	0.071 4	0.115 8	0.172 0	0.240 2	0.319 7	0.414 6	0.520 8	0.644 6	0.790 3	0.948 1	1.126 8	1.318 2	1.533 7
4.05	0.0172	0.039 5	0.072 5	0.117 5	0.174 6	0.243 9	0.324 7	0.421 1	0.528 9	0.654 7	0.802 9	0.963 2	1.144 8	1.339 4	1.558 5
4.10	0.017 5	0.040 1	0.073 6	0.119 3	0.177 3	0.247 6	0.329 6	0.427 6	0.537 1	0.664 9	0.815 4	0.978 4	1.162 8	1.360 7	1.583 3
4.15	0.017 7	0.040 7	0.074 7	0.121 1	0.179 9	0.251 3	0.334 6	0.434 1	0.545 3	0.675 1	0.828 0	0.993 5	1.180 9	1.381 9	1.608 1
4.20	0.018 0	0.041 3	0.075 8	0.122 9	0.182 6	0.255 0	0.339 6	0.440 6	0.553 5	0.685 3	0.840 5	1.008 7	1.199 0	1.403 2	1.633 0
4.25	0.018 3	0.041 9	0.076 9	0.124 6	0.185 2	0.258 8	0.344 6	0.447 1	0.561 8	0.695 6	0.853 1	1.023 8	1.217 1	1.424 5	1.657 9
4.30	0.018 5	0.042 4	0.077 9	0.126 4	0.187 9	0.262 5	0.349 6	0.453 6	0.570 0	0.705 8	0.865 7	1.039 0	1.235 2	1.445 8	1.682 8
4.35	0.018 8	0.043 0	0.079 0	0.128 2	0.190 5	0.266 2	0.354 6	0.460 1	0.578 2	0.716 0	0.878 3	1.054 2	1.253 4	1.467 2	1.707 8
4.40	0.019 0	0.043 6	0.080 1	0.130 0	0.193 2	0.269 9	0.359 6	0.466 6	0.586 5	0.726 3	0.890 9	1.069 5	1.271 6	1.488 6	1.732 8
4.45	0.019 3	0.044 2	0.081 2	0.131 8	0.195 8	0.273 7	0.364 6	0.473 1	0.594 7	0.736 6	0.903 6	1.084 7	1.289 8	1.510 0	1.757 8
4.50	0.019 5	0.044 8	0.082 3	0.133 5	0.198 5	0.277 4	0.369 6	0.479 6	0.603 0	0.746 8	0.916 2	1.099 9	1.308 0	1.531 4	1.782 8
4.55	0.019 8	0.045 4	0.083 4	0.135 3	0.201 2	0.281 1	0.374 6	0.486 2	0.611 2	0.757 1	0.928 9	1.115 2	1.326 2	1.552 8	1.807 9
4.60	0.020 1	0.046 0	0.084 5	0.137 1	0.203 8	0.284 9	0.379 6	0.492 7	0.619 5	0.767 4	0.941 5	1.130 5	1.344 4	1.574 3	1.833 0
4.65	0.020 3	0.046 6	0.085 6	0.138 9	0.206 5	0.288 6	0.384 6	0.499 3	0.627 8	0.777 7	0.954 2	1.145 8	1.362 7	1.595 8	1.858 1
4.70	0.020 6	0.047 2	0.086 7	0.140 7	0.209 2	0.292 4	0.389 6	0.505 8	0.636 0	0.788 0	0.966 9	1.161 1	1.381 0	1.617 3	1.883 3
4.75	0.020 8	0.047 8	0.087 8	0.142 5	0.211 8	0.296 1	0.394 7	0.512 4	0.644 3	0.798 3	0.979 6	1.176 4	1.399 3	1.638 9	1.908 5
4.80	0.021 1	0.048 4	0.088 9	0.144 3	0.214 5	0.299 9	0.399 7	0.518 9	0.652 6	0.808 7	0.992 3	1.191 8	1.417 6	1.660 4	1.933 7
4.85	0.021 3	0.049 0	0.090 0	0.146 1	0.217 2	0.303 6	0.404 7	0.525 5	0.660 9	0.819 0	1.005 0	1.207 1	1.435 9	1.682 0	1.958 9
4.90	0.021 6	0.049 6	0.091 1	0.147 8	0.219 9	0.307 4	0.409 8	0.532 0	0.669 2	0.829 3	1.017 8	1.222 5	1.454 3	1.703 6	1.984 1
4.95	0.021 9	0.050 2	0.092 2	0.149 6	0.222 5	0.311 2	0.414 8	0.538 6	0.677 5	0.839 7	1.030 5	1.237 8	1.472 6	1.725 2	2.009 4
5.00	0.022 1	0.050 8	0.093 3	0.151 4	0.225 2	0.314 9	0.419 8	0.545 2	0.685 9	0.850 0	1.043 3	1.253 2	1.491 0	1.746 8	2.034 7
5.05	0.022 4	0.051 4	0.094 4	0.153 2	0.227 9	0.318 7	0.424 9	0.551 8	0.694 2	0.860 4	1.056 0	1.268 6	1.509 4	1.768 4	2.060 0
5.10	0.022 6	0.052 0	0.095 5	0.155 0	0.230 6	0.322 4	0.429 9	0.558 4	0.702 5	0.870 8	1.068 8	1.284 0	1.527 8	1.790 1	2.085 3
5.15	0.022 9	0.052 6	0.096 6	0.156 8	0.233 2	0.3262	0.435 0	0.564 9	0.710 8	0.881 1	1.081 6	1.299 4	1.546 2	1.811 8	2.110 7
5.20	0.023 2	0.053 2	0.097 7	0.158 6	0.235 9	0.330 0	0.440 0	0.571 5	0.719 2	0.891 5	1.094 3	1.314 8	1.564 6	1.833 5	2.136 0
5.25	0.023 4	0.053 8	0.098 8	0.160 4	0.238 6	0.333 7	0.445 1	0.578 1	0.727 5	0.901 9	1.107 1	1.330 3	1.583 0	1.855 2	2.161 4
5.30	0.023 7	0.054 4	0.099 9	0.162 2	0.241 3	0.337 5	0.450 1	0.584 7	0.735 9	0.912 3	1.119 9	1.345 7	1.601 5	1.876 9	2.186 8
5.35	0.023 9	0.055 0	0.101 0	0.164 0	0.244 0	0.341 3	0.455 2	0.591 3	0.744 2	0.922 7	1.132 7	1.361 2	1.619 9	1.898 6	2.212 2
5.40	0.024 2	0.055 6	0.102 1	0.165 8	0.246 7	0.345 1	0.460 3	0.597 9	0.752 6	0.933 1	1.145 5	1.376 6	1.638 4	1.920 4	2.237 7
5.45	0.024 5	0.056 2	0.103 3	0.167 6	0.249 4	0.348 8	0.465 3	0.604 5	0.760 9	0.943 5	1.158 4	1.392 1	1.656 9	1.942 1	2.263 1
5.50	0.024 7	0.056 8	0.104 4	0.169 4	0.252 0	0.352 6	0.470 4	0.611 1	0.769 3	0.953 9	1.171 2	1.407 6	1.675 4	1.963 9	2.288 6
5.55	0.025 0	0.057 4	0.105 5	0.171 2	0.254 7	0.356 4	0.475 5	0.617 7	0.777 6	0.964 3	1.184 0	1.423 1	1.693 9	1.985 7	2.314 1
5.60	0.025 2	0.058 0	0.106 6	0.173 0	0.257 4	0.360 2	0.480 5	0.624 4	0.786 0	0.974 8	1.196 9	1.438 6	1.712 4	2.007 5	2.339 6
5.65	0.025 5	0.058 6	0.107 7	0.174 8	0.260 1	0.364 0	0.485 6	0.631 0	0.794 4	0.985 2	1.209 7	1.454 1	1.730 9	2.029 3	2.365 1
5.70	0.025 8	0.059 2	0.108 8	0.176											

续表

参数 q	参数 p														
	1.85	1.90	1.95	2.00	2.05	2.10	2.15	2.20	2.25	2.30	2.35	2.40	2.45	2.50	2.55
0.10	0.003 7	0.004 4	0.004 5	0.005 0	0.005 5	0.006 8	0.007 4	0.008 0	0.007 9	0.008 8	0.010 8	0.011 6	0.011 6	0.012 2	0.013 1
0.15	0.007 9	0.009 4	0.009 8	0.010 8	0.011 9	0.014 4	0.015 9	0.017 1	0.017 3	0.019 2	0.022 9	0.024 8	0.025 1	0.026 3	0.028 2
0.20	0.013 4	0.015 9	0.016 8	0.018 6	0.020 4	0.024 4	0.027 0	0.029 2	0.029 8	0.033 1	0.038 9	0.042 2	0.043 1	0.045 2	0.048 4
0.25	0.020 3	0.023 8	0.025 4	0.028 1	0.031 0	0.036 7	0.040 7	0.044 0	0.045 5	0.050 3	0.058 5	0.063 5	0.065 3	0.068 6	0.073 4
0.30	0.028 4	0.033 1	0.035 5	0.029 4	0.043 5	0.051 1	0.056 7	0.061 5	0.063 9	0.070 6	0.081 5	0.088 6	0.091 6	0.096 3	0.103 1
0.35	0.037 5	0.043 7	0.047 1	0.052 3	0.057 8	0.067 5	0.074 9	0.081 3	0.085 1	0.093 9	0.107 8	0.117 1	0.121 6	0.128 1	0.137 1
0.40	0.047 7	0.055 4	0.060 0	0.066 6	0.073 8	0.085 8	0.095 2	0.103 5	0.108 8	0.120 0	0.137 1	0.148 9	0.155 2	0.163 8	0.175 3
0.45	0.058 9	0.068 2	0.074 1	0.082 4	0.091 3	0.105 9	0.117 5	0.127 9	0.134 9	0.148 7	0.169 3	0.183 9	0.192 2	0.203 1	0.217 4
0.50	0.070 9	0.082 1	0.089 4	0.099 5	0.110 4	0.127 6	0.141 6	0.154 3	0.163 3	0.179 9	0.204 2	0.221 9	0.232 4	0.245 9	0.263 3
0.55	0.083 9	0.096 9	0.105 8	0.117 8	0.130 8	0.150 9	0.167 5	0.182 7	0.193 8	0.213 4	0.241 7	0.262 7	0.275 8	0.292 1	0.312 8
0.60	0.097 6	0.112 7	0.123 3	0.137 3	0.152 6	0.175 6	0.195 0	0.212 9	0.226 4	0.249 2	0.281 7	0.306 1	0.322 0	0.341 4	0.365 7
0.65	0.112 0	0.129 3	0.141 7	0.158 0	0.175 6	0.201 8	0.224 1	0.244 9	0.260 9	0.287 2	0.324 0	0.352 1	0.371 0	0.393 7	0.421 9
0.70	0.127 2	0.146 7	0.161 1	0.179 7	0.199 8	0.229 3	0.254 7	0.278 5	0.297 2	0.327 1	0.368 5	0.400 6	0.422 6	0.448 9	0.481 2
0.75	0.143 1	0.164 9	0.181 4	0.202 3	0.225 2	0.258 1	0.286 7	0.313 7	0.335 3	0.369 0	0.415 1	0.451 4	0.476 8	0.506 9	0.543 6
0.80	0.159 6	0.183 9	0.202 5	0.226 0	0.251 6	0.288 1	0.320 1	0.350 4	0.375 1	0.412 8	0.463 8	0.504 4	0.533 4	0.567 5	0.608 7
0.85	0.176 7	0.203 5	0.224 3	0.250 5	0.279 0	0.319 2	0.354 7	0.388 5	0.416 4	0.458 2	0.514 3	0.559 5	0.592 4	0.630 6	0.676 6
0.90	0.194 3	0.223 8	0.247 0	0.275 9	0.307 4	0.351 4	0.390 5	0.428 0	0.459 2	0.505 4	0.566 8	0.616 6	0.653 5	0.696 2	0.747 1
0.95	0.212 5	0.244 7	0.270 3	0.302 1	0.336 7	0.384 7	0.427 6	0.468 8	0.503 5	0.554 1	0.620 9	0.675 6	0.716 7	0.764 0	0.820 1
1.00	0.231 2	0.266 1	0.294 3	0.329 0	0.366 8	0.418 9	0.465 7	0.510 7	0.549 1	0.604 3	0.676 8	0.736 5	0.781 9	0.834 0	0.895 5
1.05	0.250 4	0.288 2	0.318 9	0.356 7	0.397 8	0.454 0	0.504 8	0.553 9	0.596 0	0.656 0	0.734 2	0.799 1	0.849 1	0.906 1	0.973 2
1.10	0.270 0	0.310 7	0.344 2	0.385 1	0.429 6	0.490 1	0.544 9	0.598 2	0.644 2	0.709 0	0.793 1	0.863 4	0.918 1	0.980 3	1.053 2
1.15	0.290 1	0.333 8	0.370 0	0.414 1	0.462 1	0.527 0	0.586 0	0.643 5	0.693 5	0.763 4	0.853 5	0.929 3	0.988 8	1.056 3	1.135 2
1.20	0.310 5	0.357 3	0.396 3	0.443 7	0.495 4	0.564 6	0.628 0	0.689 9	0.744 0	0.819 0	0.915 3	0.996 7	1.061 3	1.134 3	1.219 2
1.25	0.331 4	0.381 3	0.423 2	0.474 0	0.529 3	0.603 1	0.670 9	0.737 2	0.795 6	0.875 9	0.978 5	1.065 6	1.135 4	1.214 0	1.305 2
1.30	0.352 6	0.405 7	0.450 6	0.504 8	0.563 8	0.642 3	0.714 6	0.785 4	0.848 2	0.933 9	1.042 9	1.136 0	1.211 0	1.295 4	1.393 1
1.35	0.374 2	0.430 5	0.478 4	0.536 2	0.599 0	0.682 2	0.759 1	0.834 6	0.901 8	0.992 9	1.108 5	1.207 6	1.288 1	1.378 4	1.482 7
1.40	0.396 1	0.455 7	0.506 7	0.568 0	0.634 8	0.722 7	0.804 3	0.884 6	0.956 3	1.053 1	1.175 3	1.280 6	1.366 7	1.463 1	1.574 1
1.45	0.418 3	0.481 3	0.535 5	0.600 4	0.671 1	0.763 9	0.850 3	0.935 3	1.011 8	1.114 2	1.243 3	1.354 8	1.446 6	1.5492	1.667 2
1.50	0.440 8	0.507 2	0.564 6	0.633 2	0.707 9	0.805 7	0.896 9	0.986 9	1.068 1	1.176 4	1.312 3	1.430 2	1.527 9	1.6368	1.761 8
1.55	0.463 7	0.533 5	0.594 1	0.666 5	0.745 3	0.848 1	0.944 2	1.039 2	1.125 3	1.239 4	1.382 3	1.506 8	1.610 4	1.725 8	1.857 9
1.60	0.486 7	0.560 1	0.624 0	0.700 2	0.783 2	0.891 0	0.992 2	1.092 3	1.183 2	1.303 4	1.453 3	1.584 5	1.694 1	1.816 1	1.95 56
1.65	0.510 1	0.587 0	0.654 3	0.734 3	0.821 5	0.934 5	1.040 7	1.146 0	1.241 9	1.368 2	1.525 3	1.663 2	1.779 0	1.907 7	2.054 6
1.70	0.533 7	0.614 2	0.684 9	0.768 8	0.860 3	0.978 5	1.089 8	1.200 3	1.301 4	1.433 8	1.598 2	1.742 9	1.865 0	2.000 5	2.155 0
1.75	0.557 6	0.641 6	0.715 8	0.803 7	0.899 5	1.023 0	1.139 5	1.255 3	1.361 6	1.500 2	1.672 0	1.823 6	1.952 1	2.094 6	2.256 8
1.80	0.581 6	0.669 4	0.747 0	0.839 0	0.939 1	1.067 9	1.189 7	1.310 9	1.422 4	1.567 4	1.746 6	1.905 3	2.040 2	2.189 8	2.359 8
1.85	0.605 9	0.697 4	0.778 5	0.874 6	0.979 2	1.113 3	1.240 4	1.367 1	1.483 9	1.635 3	1.822 1	1.987 8	2.129 4	2.286 1	2.464 0
1.90	0.630 4	0.725 6	0.810 3	0.910 5	1.019 6	1.159 2	1.291 6	1.423 8	1.546 0	1.703 9	1.898 3	2.071 3	2.219 5	2.383 4	2.569 4
1.95	0.655 1	0.754 1	0.842 4	0.946 7	1.0603	1.205 4	1.343 3	1.481 0	1.608 7	1.773 2	1.975 3	2.155 5	2.310 5	2.481 8	2.675 9
2.00	0.680 0	0.782 8	0.874 8	0.983 3	1.101 4	1.252 0	1.395 5	1.538 8	1.672 0	1.843 1	2.053 0	2.240 6	2.402 4	2.581 2	2.783 5
2.05	0.705 1	0.811 7	0.907 4	1.020 1	1.142 9	1.299 1	1.448 0	1.597 0	1.735 8	1.913 6	2.131 4	2.326 4	2.495 2	2.681 5	2.892 2
2.10	0.730 3	0.840 8	0.940 2	1.057 2	1.184 6	1.346 4	1.501 0	1.655 8	1.800 2	1.984 8	2.210 4	2.413 0	2.588 9	2.782 8	3.001 9
2.15	0.755 8	0.870 1	0.973 3	1.094 6	1.226 7	1.394 2	1.554 4	1.715 0	1.865 1	2.056 5	2.290 1	2.500 3	2.683 3	2.884 9	3.112 5
2.20	0.781 3	0.899 6	1.006 6	1.132 2	1.269 1	1.442 3	1.608 2	1.774 6	1.930 4	2.128 7	2.370 5	2.588 3	2.7785	2.9879	3.2241
2.25	0.807 1	0.929 3	1.040 1	1.170 1	1.311 7	1.490 7	1.662 3	1.834 6	1.996 3	2.201 6	2.451 4	2.677 0	2.874 4	3.091 7	3.336 7
2.30	0.833 0	0.959 2	1.073 8	1.208 2	1.354 6	1.539 4	1.716 8	1.895 1	2.062 6	2.274 9	2.532 9	2.766 3	2.971 1	3.196 4	3.450 1
2.35	0.859 0	0.989 2	1.107 7	1.246 5	1.397 8	1.588 4	1.771 6	1.955 9	2.129 3	2.348 7	2.615 0	2.856 2	3.068 4	3.301 8	3.564 3
2.40	0.885 2	1.019 4	1.141 7	1.285 1	1.441 3	1.637 7	1.826 8	2.017 1	2.196 5	2.423 0	2.697 6	2.946 8	3.166 5	3.407 9	3.679 4
2.45	0.911 4	1.049 7	1.176 0	1.323 8	1.484 9	1.687 3	1.882 3	2.078 7	2.264 1	2.497 8	2.780 7	3.037 9	3.265 1	3.514 8	3.7953
2.50	0.937 9	1.080 2	1.210 5	1.362 8	1.528 8	1.737 2	1.938 1	2.140 6	2.332 1	2.573 0	2.864 3	3.129 6	3.364 4	3.622 3	3.911 9
2.55	0.964 4	1.110 8	1.245 1	1.402 0	1.573 0	1.787 3	1.994 2	2.202 9	2.400 5	2.648 6	2.948 4	3.221 8	3.464 3	3.730 5	4.029 3
2.60	0.991 1	1.141 6	1.279 8	1.441 3	1.617 3	1.837 6	2.050 6	2.265 5	2.469 2	2.724 7	3.033 0	3.314 6	3.564 8	3.839 4	4.147 5
2.65	1.017 8	1.172 5	1.314 8	1.480 9	1.661 9	1.888 2	2.107 3	2.328 4	2.538 3	2.801 1	3.118 0	3.407 8	3.665 8	3.948 9	4.266 3
2.70	1.044 7	1.203 6	1.349 8	1.520 6	1.706 7	1.939 1	2.164 2	2.391 6	2.607 7	2.878 0	3.203 5	3.501 6	3.767 4	4.059 0	4.3858
2.75	1.071 7	1.234 7	1.385 1	1.5604	1.751 6	1.990 1	2.221 4	2.455 1	2.677 5	2.955 2	3.289 4	3.595 8	3.869 6	4.169 7	4.505 9
2.80	1.098 8	1.266 0	1.420 4	1.600 5	1.796 8	2.041 4	2.278 8	2.518 8	2.747 6	3.032 8	3.375 3	3.690 5	3.972 2	4.281 0	4.626 7
2.85	1.126 0	1.297 4	1.455 9	1.640 7	1.842 1	2.092 9	2.336 5	2.582 9	2.818 0	3.110 7	3.462 3	3.785 6	4.075 3	4.392 8	4.748 0
2.90	1.153 2	1.328 9	1.491 5	1.681 0	1.887 6	2.144 6	2.394 4	2.647 2	2.888 7	3.189 0	3.549 4	3.881 1	4.178 9	4.505 2	4.870 0
2.95	1.180 6	1.360 5	1.527 3	1.721 5	1.933 3	2.196 5	2.452 5	2.711 8	2.959 6	3.267 6	3.636 8	3.977 1	4.283 0	4.618 0	4.992 6
3.00	1.208 0	1.392 2	1.563 2	1.762 1	1.979 1	2.248 6	2.510 8	2.776 6	3.030 9	3.346 5	3.724 6	4.073 4	4.387 5	4.731 4	5.115 7
3.05	1.235 6	1.424 0	1.599 1	1.802 9	2.025 1	2.300 8	2.569 4	2.841 7	3.102 4	3.425 7	3.812 8	4.170 2	4.492 5	4.845 2	5.239 3
3.10	1.263 2	1.455 9	1.635 2	1.843 8	2.071 3	2.353 2	2.628 1	2.906 9	3.174 2	3.505 2	3.901 2	4.267 3	4.597 8	4.959 6	5.363 5
3.15	1.290 9	1.487 9	1.671 4	1.884 8	2.117 6	2.405 8	2.687 0	2.972 4	3.246 3	3.585 0	3.990 0	4.364 8	4.703 6	5.074 3	5.488 2

续表

参数 q	参数 p														
	1.85	1.90	1.95	2.00	2.05	2.10	2.15	2.20	2.25	2.30	2.35	2.40	2.45	2.50	2.55
3.20	1.318 6	1.520 0	1.707 8	1.926 0	2.164 0	2.458 6	2.746 2	3.038 2	3.318 6	3.665 1	4.079 2	4.462 6	4.809 8	5.189 5	5.613 3
3.25	1.346 5	1.552 1	1.744 2	1.967 2	2.210 6	2.511 5	2.805 5	3.104 1	3.391 1	3.745 4	4.168 6	4.560 8	4.916 3	5.305 2	5.739 0
3.30	1.374 4	1.584 4	1.780 7	2.008 6	2.257 3	2.564 6	2.865 0	3.170 2	3.463 9	3.826 0	4.258 3	4.659 3	5.023 3	5.421 3	5.865 1
3.35	1.402 4	1.616 7	1.817 3	2.050 1	2.304 1	2.617 8	2.924 6	3.236 6	3.536 9	3.906 9	4.348 3	4.758 1	5.130 6	5.537 7	5.991 6
3.40	1.430 4	1.649 1	1.854 0	2.091 7	2.351 1	2.6712	2.984 5	3.303 1	3.610 1	3.988 0	4.438 5	4.857 3	5.238 2	5.654 6	6.118 6
3.45	1.458 5	1.681 6	1.890 8	2.133 4	2.398 1	2.724 7	3.044 5	3.369 8	3.683 5	4.069 4	4.529 1	4.956 7	5.346 2	5.771 8	6.246 0
3.50	1.486 7	1.714 2	1.927 6	2.175 2	2.445 3	2.778 3	3.104 6	3.436 7	3.757 1	4.150 9	4.619 9	5.056 5	5.454 5	5.889 4	6.373 9
3.55	1.514 9	1.746 8	1.964 6	2.217 1	2.492 6	2.832 1	3.164 9	3.503 7	3.830 9	4.232 7	4.710 9	5.156 5	5.563 1	6.007 4	6.502 1
3.60	1.543 2	1.779 5	2.001 6	2.259 1	2.540 1	2.886 0	3.225 3	3.570 9	3.904 9	4.314 8	4.802 2	5.256 8	5.672 1	6.125 7	6.630 7
3.65	1.571 5	1.812 3	2.038 7	2.301 2	2.587 6	2.940 0	3.285 9	3.638 3	3.979 1	4.397 0	4.893 8	5.357 4	5.781 3	6.244 3	6.759 7
3.70	1.599 9	1.845 1	2.075 9	2.343 3	2.635 2	2.9942	3.346 6	3.705 9	4.053 5	4.479 4	4.985 5	5.458 2	5.890 8	6.363 3	6.889 0
3.75	1.628 3	1.878 0	2.113 2	2.385 6	2.682 9	3.048 4	3.407 5	3.773 6	4.128 0	4.562 1	5.077 5	5.559 3	6.000 7	6.482 6	7.018 8
3.80	1.656 8	1.910 9	2.150 5	2.427 9	2.730 8	3.102 8	3.468 5	3.841 4	4.202 7	4.644 9	5.169 8	5.660 6	6.110 8	6.602 2	7.148 8
3.85	1.685 4	1.943 9	2.187 9	2.470 4	2.778 7	3.157 3	3.529 6	3.909 4	4.277 6	4.727 9	5.262 2	5.762 2	6.221 1	6.722 1	7.279 2
3.90	1.714 0	1.977 0	2.225 4	2.512 9	2.826 7	3.211 9	3.590 8	3.977 5	4.352 6	4.811 1	5.354 8	5.864 0	6.331 8	6.842 3	7.409 9
3.95	1.742 6	2.010 1	2.262 9	2.555 4	2.874 8	3.266 6	3.652 2	4.045 8	4.427 8	4.894 5	5.447 6	5.966 0	6.442 6	6.962 8	7.541 0
4.00	1.771 3	2.043 3	2.300 5	2.598 1	2.923 0	3.321 3	3.713 6	4.114 2	4.503 2	4.978 0	5.540 7	6.068 3	6.553 8	7.083 6	7.672 3
4.05	1.800 2	2.076 5	2.338 1	2.640 8	2.971 2	3.376 2	3.775 2	4.182 7	4.578 7	5.061 7	5.633 9	6.170 8	6.665 1	7.204 6	7.804 0
4.10	1.828 8	2.109 8	2.375 8	2.683 6	3.019 6	3.431 2	3.836 9	4.251 4	4.654 3	5.145 6	5.727 3	6.273 4	6.776 7	7.325 9	7.935 9
4.15	1.857 6	2.143 1	2.413 6	2.726 5	3.068 0	3.486 3	3.898 7	4.320 1	4.730 1	5.229 6	5.820 9	6.376 3	6.888 6	7.447 4	8.068 2
4.20	1.886 4	2.176 4	2.451 4	2.769 4	3.116 5	3.541 4	3.960 6	4.389 0	4.806 0	5.313 8	5.914 6	6.479 4	7.000 6	7.569 2	8.200 7
4.25	1.915 3	2.209 9	2.489 3	2.812 4	3.165 1	3.596 7	4.022 6	4.458 0	4.882 0	5.398 1	6.008 5	6.582 6	7.112 9	7.691 3	8.333 5
4.30	1.944 2	2.243 3	2.527 3	2.855 4	3.213 8	3.652 0	4.084 6	4.527 1	4.958 2	5.482 6	6.102 6	6.686 1	7.225 4	7.813 6	8.466 5
4.35	1.973 2	2.276 8	2.565 2	2.898 5	3.262 5	3.707 4	4.146 8	4.596 4	5.034 5	5.567 2	6.196 9	6.789 7	7.338 1	7.936 1	8.599 8
4.40	2.002 2	2.310 4	2.603 3	2.941 7	3.311 3	3.762 9	4.209 1	4.665 7	5.110 9	5.652 0	6.291 3	6.893 5	7.451 0	8.058 8	8.733 4
4.45	2.031 2	2.343 9	2.641 4	2.984 9	3.360 1	3.818 5	4.271 5	4.735 1	5.187 4	5.736 9	6.385 8	6.997 5	7.564 0	8.181 8	8.867 2
4.50	2.060 2	2.377 6	2.679 5	3.028 2	3.409 1	3.874 1	4.333 9	4.804 7	5.264 1	5.821 9	6.480 2	7.101 7	7.677 3	8.304 9	9.001 2
4.55	2.089 3	2.411 2	2.717 7	3.071 5	3.458 0	3.929 8	4.396 5	4.874 3	5.340 8	5.907 1	6.575 4	7.206 0	7.790 8	8.428 3	9.135 5
4.60	2.118 5	2.444 9	2.755 9	3.114 9	3.507 1	3.985 6	4.459 1	4.944 0	5.417 7	5.9923	6.670 4	7.310 5	7.904 4	8.551 9	9.270 0
4.65	2.147 6	2.478 7	2.794 1	3.158 4	3.556 2	4.041 5	4.521 8	5.013 8	5.494 6	6.077 7	6.765 5	7.415 1	8.018 2	8.675 7	9.404 7
4.70	2.176 8	2.512 4	2.832 4	3.201 8	3.605 4	4.097 4	4.584 6	5.083 7	5.571 7	6.163 3	6.860 8	7.519 9	8.132 2	8.799 6	9.539 6
4.75	2.206 0	2.546 2	2.870 8	3.245 4	3.654 6	4.153 4	4.647 4	5.153 7	5.648 9	6.248 9	6.956 2	7.624 8	8.246 4	8.923 8	9.674 8
4.80	2.235 3	2.580 1	2.909 2	3.289 0	3.703 9	4.209 5	4.710 3	5.223 8	5.726 2	6.334 6	7.051 7	7.729 9	8.360 7	9.048 1	9.810 1
4.85	2.264 5	2.614 0	2.947 6	3.332 6	3.753 2	4.265 6	4.773 3	5.294 0	5.803 5	6.420 5	7.147 4	7.835 1	8.475 1	9.172 6	9.945 7
4.90	2.293 8	2.647 9	2.986 1	3.376 3	3.802 6	4.321 8	4.836 4	5.364 2	5.881 0	6.506 4	7.243 1	7.940 5	8.589 8	9.297 3	10.081 4
4.95	2.323 1	2.681 8	3.024 6	3.420 0	3.852 0	4.378 0	4.899 6	5.434 5	5.958 6	6.592 5	7.339 0	8.046 0	8.704 5	9.422 2	10.217 4
5.00	2.352 5	2.715 8	3.063 1	3.463 7	3.901 5	4.434 3	4.962 8	5.505 0	6.036 2	6.678 7	7.435 0	8.151 6	8.819 5	9.547 2	10.353 5
5.05	2.381 9	2.749 8	3.101 7	3.507 5	3.951 0	4.490 7	5.026 0	5.575 4	6.113 9	6.764 9	7.531 1	8.257 3	8.934 5	9.6724	10.489 8
5.10	2.411 2	2.783 8	3.140 3	3.551 4	4.000 6	4.547 1	5.089 4	5.646 0	6.191 7	6.851 3	7.627 4	8.363 2	9.049 8	9.797 8	10.626 3
5.15	2.440 7	2.817 8	3.178 9	3.595 3	4.050 2	4.603 5	5.152 8	5.716 6	6.2696	6.937 7	7.723 7	8.469 2	9.165 1	9.923 3	10.763 0
5.20	2.470 1	2.851 9	3.217 6	3.639 2	4.099 9	4.660 0	5.216 2	5.787 3	6.347 6	7.024 3	7.820 1	8.575 3	9.280 6	10.048 9	10.899 8
5.25	2.499 6	2.886 0	3.256 3	3.683 1	4.149 6	4.716 6	5.279 8	5.858 1	6.425 7	7.110 9	7.916 7	8.681 5	9.396 2	10.174 7	11.036 8
5.30	2.529 1	2.920 2	3.295 0	3.727 1	4.199 4	4.773 2	5.343 3	5.928 9	6.503 8	7.197 7	8.013 3	8.787 9	9.512 0	10.300 7	11.174 0
5.35	2.558 6	2.954 3	3.333 8	3.771 2	4.249 2	4.829 9	5.407 0	5.999 8	6.582 0	7.284 5	8.110 1	8.894 3	9.627 8	10.426 8	11.311 3
5.40	2.588 1	2.988 5	3.372 6	3.815 2	4.299 0	4.886 6	5.4707	6.070 8	6.660 3	7.371 4	8.206 9	9.000 9	9.743 8	10.553 0	11.448 8
5.45	2.617 6	3.022 7	3.411 4	3.859 3	4.348 9	4.943 4	5.534 4	6.141 8	6.738 6	7.458 3	8.303 8	9.107 6	9.859 9	10.679 4	11.586 4
5.50	2.647 2	3.056 9	3.450 3	3.903 5	4.398 8	5.000 2	5.598 2	6.212 9	6.817 1	7.545 4	8.400 8	9.214 3	9.976 2	10.805 9	11.724 2
5.55	2.676 8	3.091 2	3.489 1	3.947 6	4.448 8	5.057 0	5.662 1	6.284 0	6.895 6	7.632 5	8.498 0	9.321 2	10.092 5	10.932 5	11.862 1
5.60	2.706 4	3.125 5	3.528 1	3.991 8	4.498 8	5.113 9	5.726 0	6.355 2	6.974 1	7.719 7	8.595 2	9.428 2	10.209 0	11.059 2	12.000 2
5.65	2.736 0	3.159 8	3.567 0	4.036 0	4.548 8	5.170 8	5.789 9	6.426 5	7.052 7	7.807 0	8.692 4	9.535 2	10.325 5	11.186 1	12.138 4
5.70	2.765 7	3.194 1	3.605 9	4.080 3	4.598 9	5.227 8	5.853 9	6.497 8	7.131 4	7.894 4	8.789 8	9.642 4	10.442 2	11.313 1	12.276 7
5.75	2.795 3	3.228 4	3.644 9	4.124 6	4.649 0	5.284 8	5.917 9	6.569 1	7.210 2	7.981 8	8.887 2	9.749 7	10.559 0	11.440 2	12.415 2
5.80	2.825 0	3.262 8	3.683 9	4.168 9	4.699 1	5.341 9	5.982 0	6.640 6	7.289 0	8.069 3	8.984 8	9.857 0	10.675 9	11.567 4	12.553 8
5.85	2.854 7	3.297 2	3.723 0	4.213 3	4.749 3	5.399 0	6.046 2	6.712 0	7.367 9	8.156 9	9.082 4	9.964 4	10.792 8	11.694 8	12.692 5
5.90	2.884 4	3.331 6	3.762 0	4.257 6	4.799 5	5.456 1	6.110 4	6.783 6	7.446 8	8.244 5	9.180 1	10.072 0	10.909 1	11.822 2	12.831 4
5.95	2.914 1	3.366 0	3.801 1	4.302 0	4.849 7	5.513 3	6.174 6	6.855 1	7.525 8	8.332 2	9.277 8	10.179 6	11.027 1	11.949 8	12.970 3
6.00	2.943 9	3.400 4	3.840 2	4.346 5	4.900 0	5.570 5	6.238 8	6.926 8	7.604 8	8.420 0	9.375 6	10.287 3	11.144 4	12.077 5	13.109 4
6.05	2.973 6	3.434 9	3.879 3	4.390 9	4.950 3	5.627 8	6.303 1	6.998 4	7.683 9	8.507 8	9.473 5	10.395 0	11.261 7	12.205 2	13.248 6
6.10	3.003 4	3.469 4	3.918 5	4.435 4	5.000 6	5.685 0	6.367 5	7.070 1	7.763 1	8.595 7	9.571 5	10.502 9	11.379 2	12.333 1	13.388 0
6.15	3.033 2	3.503 9	3.957 6	4.479 9	5.050 9	5.742 3	6.431 9	7.141 9	7.842 3	8.683 7	9.669 5	10.610 8	11.496 7	12.461 1	13.527 4
6.20	3.063 0	3.538 4	3.996 8	4.524 4	5.101 3	5.799 7	6.496 3	7.213 7	7.921 5	8.771 7	9.767 7	10.718 8	11.614 3	12.589 2	13.667 0
6.25	3.092 8	3.572 9	4.036 0	4.569 0	5.151 7	5.857 1	6.560 8	7.285 6	8.000 8	8.859 7	9.865 8	10			

4 应用实例

设空心圆柱线圈的参数为:内半径 $R_1 = 0.03\text{ m}$, 外半径 $R_2 = 0.05\text{ m}$, 线圈长度 $D = 0.10\text{ m}$, 匝流密度 $n_c = 5 \times 10^5$, 则形状参数 $p = R_2/R_1 = 1.67, q = D/R_1 = 3.33$. 查表 1 知函数表中没有相对应的数值. 利用插值公式(7)可求得 $T \approx 0.795\ 01$. 把以上参数代入式(5)可得到电感值为 $38.133\ 7\text{ mH}$, 而它的准确值是 $38.120\ 2\text{ mH}$, 误差为 0.036% . 而利用参考文献[1]所给表达式求得电感为 37.92 mH , 误差为 0.525% , 可见, 利用本文的函数表计算电感时精度要高一些.

5 结论

本文利用空心圆柱线圈计算电感的一般解析

表达式给出了计算函数 T 的函数表, 与 $\Pi\cdot\text{JI}\cdot$ 卡兰塔罗夫所给表达式相比, 利用本文所给表达式和函数表近似地求解线圈电感时, 可以得到精度更高的结果.

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Inductance Tables of Air-cored Cylindrical Coil

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Abstract : In this paper , the boundary value problem of the magnetic vector potential is derived by use of the character of the axisymmetrical electromagnetic field . Through solving the boundary value problem , the magnetic vector potential is gained . And then , the expression for calculating inductance of air-cored cylindrical coil is derived in this paper by using vector potential . And the function tables of function T in the calculating expression are given for the convenience of the calculation of inductance while the precision is not strictly required . Finally , the use of function tables is introduced with an example . At the same time , the example shows that the result has higher precision when the inductance of coil is calculated by use of the function tables provided in this paper .

Key words : air-cored cylindrical coil ; inductance ; vector potential ; function table