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一般 CDF 方程一种不变性

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摘要 本文给出一般 CDF 方程的一种不变性。据此，我们可以仅通过积分的方法，从一般 CDF 方程的已知解得到新的解。本文所得结果概括了文 [3] 的结论。

关键词 CDF 方程 Backlund 变换 MKdV 方程

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MKdV 方程

$$q_t = q_{xxx} + 6q^2 q_x \tag{1}$$

的 Riccati 形式的 Lax 对可化为

$$\varphi_x = -(q + \xi\sin 2\varphi)$$
$$\varphi_t = \varphi_{xxx} + 2\varphi_x^3 + 6\xi\varphi_x\sin^2 2\varphi \tag{2}$$

其中 ξ 是一个任意常数[1]，方程 (2) 称为 CDF (Calogero－Degasperies－Fordy) 方程[2]. 文 [3] 给出了 CDF 方程的一种不变性。本文讨论一般 CDF 方程

$$\varphi_t = \varphi_{xxx} + 2\varphi_x^3 + 6\varphi_x(\xi\sin 2\varphi + \eta\cos\varphi)^2 \tag{3}$$

其中 ξ 和 η 是两个任意常数，从一般 CDF 方程的 Backlund 变换出发，给出一般 CDF 方程的一种不变性。根据这一不变性，我们可以仅通过积分的方法，从一般 CDF 方程的已知解得到新的解。

1 一般 CDF 方程的一种不变性

引理 1 一般 CDF 方程 (3) 的 Backlund 变换 (BT) 是

$$\overline{\varphi}_x = \varphi_x + \xi\sin 2\varphi + \eta\cos 2\varphi - \xi\sin 2\overline{\varphi} - \eta\cos 2\overline{\varphi} \tag{4}$$

$$\overline{\varphi}_t = \varphi_t + R \tag{5}$$

其中

$$\begin{aligned} R = {} & 2[\varphi_{xx} - 2\varphi_x(\xi\cos 2\overline{\varphi} - \eta\sin 2\overline{\varphi})](\xi\cos 2\varphi - \eta\sin 2\varphi - \xi\cos 2\overline{\varphi} + \eta\sin 2\overline{\varphi}) \\ & + 2[\varphi_x^2 - 2\varphi_x(\xi\sin 2\overline{\varphi} + \eta\cos 2\overline{\varphi}) + (\xi\sin 2\varphi + \eta\cos 2\varphi)^2 + 2(\xi + \eta^2)] \\ & (\xi\sin 2\varphi + \eta\cos 2\varphi - \xi\sin 2\overline{\varphi} - \eta\cos 2\overline{\varphi}). \end{aligned}$$

证明：设 φ 是一般 CDF 方程 (3) 的一个解，由 (4) 可得

$$\overline{\varphi}_{xx} = \varphi_{xx} + 2\varphi_x(\xi\cos 2\varphi - \eta\sin 2\varphi) - 2\overline{\varphi}_x(\xi\cos 2\overline{\varphi} - \eta\sin 2\overline{\varphi})$$

$$\begin{aligned} \overline{\varphi}_{xxx} = {} & \varphi_{xxx} + 2\varphi_{xx}(\xi\cos 2\varphi - \eta\sin 2\varphi) - 4\varphi_x^2(\xi\sin 2\varphi + \eta\cos 2\varphi) - 2[\varphi_{xx} + \\ & 2\varphi_x(\xi\cos 2\varphi - \eta\sin 2\varphi) - 2\overline{\varphi}_x(\xi\cos 2\overline{\varphi} - \eta\sin 2\overline{\varphi})](\xi\cos 2\overline{\varphi} - \eta\sin 2\overline{\varphi}) \end{aligned}$$

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$$
\begin{aligned}
&+4(\varnothing_x+\xi\sin 2\varnothing+\eta\cos 2\varnothing-\xi\sin 2\overline{\varnothing}-\eta\cos 2\overline{\varnothing})^2(\xi\sin 2\overline{\varnothing}+\eta\cos 2\overline{\varnothing})\\
=&\varnothing_{xxx}+2\varnothing_{xx}(\xi\cos 2\varnothing-\eta\sin 2\varnothing-\xi\cos 2\overline{\varnothing}+\eta\sin 2\overline{\varnothing})\\
&-4\varnothing_x^2(\xi\sin 2\varnothing+\eta\cos 2\varnothing-\xi\sin 2\overline{\varnothing}-\eta\cos 2\overline{\varnothing})\\
&8\varnothing_x(\xi\sin 2\varnothing+\eta\cos 2\varnothing-\xi\sin 2\overline{\varnothing}-\eta\cos 2\overline{\varnothing})(\xi\sin 2\overline{\varnothing}+\eta\cos 2\overline{\varnothing})\\
&-4\varnothing_x(\xi\cos 2\varnothing-\eta\sin 2\varnothing-\xi\cos 2\overline{\varnothing}+\eta\sin 2\overline{\varnothing})(\xi\cos 2\overline{\varnothing}-\eta\sin 2\overline{\varnothing})\\
&+4(\xi\sin 2\varnothing+\eta\cos 2\varnothing-\xi\sin 2\overline{\varnothing}-\eta\cos 2\overline{\varnothing})(\xi\cos 2\overline{\varnothing}-\eta\, sinw\, 2\overline{\varnothing})^2\\
&+4(\xi\sin 2\varnothing+\eta\cos 2\varnothing-\xi\sin 2\overline{\varnothing}-\eta\cos 2\overline{\varnothing})^2(\xi\sin 2\overline{\varnothing}+\eta\cos 2\overline{\varnothing}).
\end{aligned}
$$

和$2\overline{\varnothing}_x^3+6\overline{\varnothing}_x(\xi\sin 2\overline{\varnothing}+\eta\cos 2\overline{\varnothing})^2$

$$
\begin{aligned}
=&2\varnothing_x^3+6\varnothing_x(\xi\sin 2\varnothing+\eta\cos 2\varnothing)^2+6\varnothing_x^2(\xi\sin 2\varnothing+\eta\cos 2\varnothing-\xi\sin 2\overline{\varnothing}-\eta\cos 2\overline{\varnothing})\\
&-12\varnothing_x(\xi\sin 2\varnothing+\eta\cos 2\varnothing-\xi\sin 2\overline{\varnothing}-\eta\cos 2\overline{\varnothing})(\xi\sin 2\overline{\varnothing}+\eta\cos 2\overline{\varnothing})\\
&+6(\xi\sin 2\varnothing+\eta\cos 2\varnothing-\xi\sin 2\overline{\varnothing}-\eta\cos 2\overline{\varnothing})(\xi\sin 2\overline{\varnothing}+\eta\cos 2\overline{\varnothing})^2\\
&+2(\xi\sin 2\varnothing+\eta\cos 2\varnothing-\xi\sin 2\overline{\varnothing}-\eta\cos 2\overline{\varnothing})^3.
\end{aligned}
$$

那么

$$
\begin{aligned}
&\overline{\varnothing}_{xxx}+2\overline{\varnothing}_x^3+6\overline{\varnothing}_x(\xi\sin 2\overline{\varnothing}+\eta\cos 2\overline{\varnothing})^2\\
=&\varnothing_{xxx}+2\varnothing_x^3+6\varnothing_x(\xi\sin 2\varnothing+\eta\cos 2\overline{\varnothing})^2+R\\
=&\varnothing_t+R.
\end{aligned}
$$

因此，由（5）可得$\overline{\varnothing}_{xxx}+2\overline{\varnothing}_x^3+6\overline{\varnothing}_x(\xi\sin 2\overline{\varnothing}+\eta\cos 2\overline{\varnothing})^2=\overline{\varnothing}$

另一方面，如果$\overline{\varnothing}$是一般 CDF 方程（3）的一个解，那么$\varnothing$也是一个解。

因此，按照 Backlund 变换（BT）的定义（参看［4，p．154］，关系式（4）和（5）是一般 CDF 方程的一个 BT．引理证毕。

按照 BT 纲领，如果已知某非线性微分方程的一个解，我们可以利用 BT 关系找出该微分方程的新解。

在下面，我们用$\int f dx$表示 f 的任意取定的一个原函数。

引理 2 如果$\varnothing$是一般 CDF 方程（3）的一个解，那么

$$
\begin{aligned}
C\equiv&-\frac{1}{2}\left[\int(\xi\cos 2\varnothing-\eta\sin 2\varnothing)dx\right]_t-\varnothing_{xx}(\xi\sin 2\varnothing+\eta\cos 2\varnothing)+\varnothing_x^2(\xi\cos 2\varnothing-\eta\sin 2\varnothing)\\
&[(\xi\sin 2\varnothing+\eta\cos 2\varnothing)^2+2(\xi+\eta^2)](\xi\cos 2\varnothing-\eta\sin 2\varnothing)
\end{aligned}
$$

和

$$
\begin{aligned}
h\equiv&-2\left[\int(\xi\sin 2\varnothing+\eta\cos 2\varnothing)e^{-2\int(\xi\cos 2\varnothing-\eta\sin 2\varnothing)dx}dx\right]_t\\
&+8c\int(\xi\sin 2\varnothing+\eta\cos 2\varnothing)e^{-2\int(\xi\cos 2\varnothing-\eta\sin 2\varnothing)dx}dx\\
&+4[\varnothing_{xx}+2\varnothing_x(\xi\cos 2\varnothing-\eta\sin 2\varnothing)](\xi\cos 2\varnothing-\eta\sin 2\varnothing)e^{-2\int(\xi\cos 2\varnothing-\eta\sin 2\varnothing)dx}\\
&+4[\varnothing_x^2+2\varnothing_x(\xi\sin 2\varnothing+\eta\cos 2\varnothing)+(\xi\sin 2\varnothing+\eta\cos 2\varnothing)^2\\
&+2(\xi+\eta^2)](\xi\sin 2\varnothing+\eta\cos 2\varnothing)e^{-2\int(\xi\cos 2\varnothing-\eta\sin 2\varnothing)dx}
\end{aligned}
$$

是仅含 t 的函数（即，$c_x=h_x=0$）．

证明：如果$\varnothing$是一般 CDF 方程（3）的一个解，那么

$$C_x = (\xi sin2\varnothing + \eta cos2\varnothing)[\varnothing_t - \varnothing_{xxx} - 2\varnothing_x^3 - 6\varnothing_x(\xi sin2\varnothing + \eta cos2\varnothing)^2] = 0,$$

$$h_x = (\xi cos2\varnothing - \eta sin2\varnothing)[\varnothing_{xxx} + 2\varnothing_x^3 + 6\varnothing_x(\xi sin2\varnothing + \eta cos2\varnothing)^2 - \varnothing_t]e^{-2\int(\xi cos\varnothing - \eta sin2\varnothing)dx}$$
$$+ 8(\xi sin2\varnothing + \eta cos2\varnothing)[c + \frac{1}{2}(\int(\xi cos2\varnothing - \eta sin2\varnothing)dx)_t + \varnothing_{xx}(\xi sin2\varnothing$$
$$+ \eta cos2\varnothing) - \varnothing_x^2(\xi cos2\varnothing - \eta sin2\varnothing) - ((\xi sin2\varnothing + \eta cos2\varnothing)^2 + 2(\xi$$
$$+ \eta^2))(\xi cos2\varnothing - \eta sin2\varnothing)]e^{-2\int(\xi cos2\varnothing - \eta sin2\varnothing)dx} = 0。$$

引理证毕。

定理 3　如果$\varnothing$是一般 CDF 方程（3）的一个解，那么

$$\overline{\varnothing} = arctan(tan\varnothing + Y) \tag{6}$$

也是（3）的一个解，其中

$$Y = sec^2\varnothing e^{-2\int(\xi cos2\varnothing - \eta sin2\varnothing)dx}/Q \tag{7}$$

$$Q = -tan\varnothing e^{-2\int(\xi cos2\varnothing - \eta sin2\varnothing)dx} - 2\int(\xi sin2\varnothing + \eta cos2\varnothing)e^{-2\int(\xi cos2\varnothing - \eta sin2\varnothing)dx}dx + \varepsilon \tag{8}$$

$$\varepsilon = e^{4\int c(t)dt}(\alpha - \int h(t)e^{-4\int c(t)dt}dt) \tag{9}$$

α是一个任意常数。(即，$\varepsilon' = 4c(t)\varepsilon - h(t)$)

证明：我们直接验证$\varnothing$和$\overline{\varnothing}$满足 BT 关系（4）和（5）。

由（6），（7）和（8），我们有

$$\overline{\varnothing}_x sec^2\overline{\varnothing} = \varnothing_x sec^2\varnothing + Y_x,$$

$$Y_x = 2[tan\varnothing(\varnothing_x + \xi sin2\varnothing + \eta cos2\varnothing + \eta) - \xi]Y$$
$$+ (\varnothing_x + \xi sin2\varnothing + \eta cos2\varnothing + \eta)Y^2,$$

$$sec^2\overline{\varnothing}(\overline{\varnothing}_x + \xi sin2\overline{\varnothing} + \eta cos2\overline{\varnothing}) = sec^2\overline{\varnothing}(\varnothing_x + \xi sin2\varnothing + \eta cos2\varnothing).$$

因此，（4）式成立，下面，我们仅需证明（5）式成立。

由于

$$\overline{\varnothing}_t = \varnothing_t sec^2\varnothing/sec^2\overline{\varnothing} + Y_t/sec^{2-}\overline{\varnothing}$$

$$sec^2\overline{\varnothing} = 1 + (tan\varnothing + Y)^2 = sec^2\varnothing + 2Ytan\varnothing + Y^2,$$

（5）成立的充分必要条件是

$$-(2Ytan\varnothing + Y^2)\varnothing_t + Y_t = Rsec^2\overline{\varnothing}. \tag{10}$$

由（6），（7）和（8），

$$Y_t = 2Y\varnothing_t tan\varnothing - 2Y[\int(\xi cos2\varnothing - \eta sin2\varnothing)dx]_t - YQ_t/Q,$$

$$Q_t = [-\varnothing_t sec^2\varnothing + 2tan\varnothing(\int(\xi cos2\varnothing - \eta sin2\varnothing)dx)_t]e^{-2\int(\xi cos2\varnothing - \eta sin2\varnothing)dx}$$
$$- 2[\int(\xi sin2\varnothing + \eta cos2\varnothing)e^{-2\int(\xi cos2\varnothing - \eta sin2\varnothing)dx}dx]_t + \varepsilon',$$

我们有

$$Y_t = \varnothing_t(2Ytan\varnothing + Y^2) - 2Y(\int(\xi cos2\varnothing - \eta sin2\varnothing)dx)_t$$

$$- Y^2 \sin 2\varnothing \left(\int (\xi\cos 2\varnothing - \eta\sin 2\varnothing)\, dx\right)_t + 2Y\left[\int (\xi\sin 2\varnothing + \eta\cos 2\varnothing)\, e^{-2\int(\xi\cos 2\varnothing - \eta\sin 2\varnothing)dx} dx\right]_t / Q - Y\varepsilon'/Q.$$

那么 (10) 化为

$$- 2Y\left[\int (\xi\cos 2\varnothing - \eta\sin 2\varnothing)\, dx\right]_t - Y^2 \sin 2\varnothing\left(\int (\xi\cos 2\varnothing - \eta\sin 2\varnothing)\, dx\right)_t + 2Y\left[\int (\xi\sin 2\varnothing + \eta\cos 2\varnothing)\, e^{-2\int(\xi\cos 2\varnothing - \eta\sin 2\varnothing)dx} dx\right]_t / Q - Y\varepsilon'/Q = R\sec^2\overline{\varnothing} \tag{11}$$

将

$$\cos 2\varnothing - \cos 2\overline{\varnothing} = 2Y(2\tan\varnothing + Y)/\sec^2\varnothing\sec^2\overline{\varnothing},$$
$$\sin 2\varnothing - \sin 2\overline{\varnothing} = 2Y(\tan^2\varnothing + Y\tan\varnothing - 1)/\sec^2\varnothing\sec^2\overline{\varnothing}$$

代入 R 的表达式，(11) 化为

$$\begin{aligned} & - 2\left[\int (\xi\cos 2\varnothing - \eta\sin 2\varnothing)\, dx\right]_t - Y\sin 2\varnothing\left(\int (\xi\cos 2\varnothing - \eta\sin 2\varnothing)\, dx\right)_t \\ & + 2\left[\int (\xi\sin 2\varnothing + \eta\cos 2\varnothing)\, e^{-2\int(\xi\cos 2\varnothing - \eta\sin 2\varnothing)dx} dx\right]_t / Q - \varepsilon'/Q \\ = & 4\cos^2\varnothing\big[(\varnothing_{xx} + 2\varnothing_x(\xi\cos 2\varnothing - \eta\sin 2\varnothing))(\xi(2\tan\varnothing + Y) - \eta(\tan^2\varnothing + Y\tan\varnothing - 1)) + (\varnothing_x^2 + 2\varnothing_x(\xi\sin 2\varnothing \\ & + \eta\cos 2\varnothing) + (\xi\sin 2\varnothing + \eta\cos 2\varnothing)^2 + 2(\xi + \eta^2)))(\xi(\tan^2\varnothing + Y\tan\varnothing - 1) + \eta(2\tan\varnothing + Y))\big]. \end{aligned} \tag{12}$$

由于

$$\begin{aligned} & - 2\left[\int (\xi\cos 2\varnothing - \eta\sin 2\varnothing)\, dx\right]_t - 4\cos^2\varnothing[\varnothing_{xx} + 2\varnothing_x(\xi\cos 2\varnothing - \eta\sin 2\varnothing)][2\xi\tan\varnothing - \eta(\tan^2\varnothing - 1)] - 4\cos^2\varnothing[\varnothing_x^2 + 2\varnothing_x(\xi\sin 2\varnothing \\ & + \eta\cos 2\varnothing) + (\xi\sin 2\varnothing + \eta\cos 2\varnothing)^2 + 2(\xi + \eta^2)][\xi(\tan^2\varnothing - 1) + 2\eta\tan\varnothing] \\ & = 4c(t), \end{aligned}$$

$$\begin{aligned} & - \sin 2\varnothing\left[\int (\xi\cos 2\varnothing - \eta\sin 2\varnothing)\, dx\right]_t - 4\cos^2\varnothing[\varnothing_{xx} + 2\varnothing_x(\xi\cos 2\varnothing - \eta\sin 2\varnothing)] \\ & [\xi - \eta\tan\varnothing] - 4\cos^2\varnothing[\varnothing_x^2 + 2\varnothing_x(\xi\sin 2\varnothing + \eta\cos 2\varnothing) + (\xi\sin 2\varnothing \\ & + \eta\cos 2\varnothing)^2 + 2(\xi + \eta^2)][\xi\tan\varnothing + \eta] \\ & = H\cos^2\varnothing, \end{aligned}$$

其中

$$\begin{aligned} H = & - 4[\varnothing_{xx} + 2\varnothing_x(\xi\cos 2\varnothing - \eta\sin 2\varnothing)](\xi\cos 2\varnothing - \eta\sin 2\varnothing) \\ & - 4[\varnothing_x^2 + 2\varnothing_x(\xi\sin 2\varnothing + \eta\cos 2\varnothing) + (\xi\sin 2\varnothing + \eta\cos 2\varnothing)^2 \\ & + 2(\xi + \eta^2)](\xi\sin 2\varnothing + \eta\cos 2\varnothing) + 4c(t)\tan\varnothing. \end{aligned}$$

因此，(12) 化为

$$\begin{aligned} & 4c(t) + H e^{-2\int(\xi\cos 2\varnothing - \eta\sin 2\varnothing)dx}/Q \\ & + 2\left[\int (\xi\sin 2\varnothing + \eta\cos 2\varnothing)\, e^{-2\int(\xi\cos 2\varnothing - \eta\sin 2\varnothing)dx} dx\right]_t / Q - \varepsilon'/Q = 0. \end{aligned} \tag{13}$$

将 (8) 代入 (13)，我们有

$$4c(t)\varepsilon-\varepsilon'+(H-4c(t)\tan\varnothing)e^{-2\int(\xi\cos2\varnothing-\eta\sin2\varnothing)dx}$$
$$-8c(t)\int(\xi\sin2\varnothing+\eta\cos2\varnothing)e^{-2\int(\xi\cos2\varnothing-\eta\sin2\varnothing)dx}dx$$
$$+2\left[\int(\xi\sin2\varnothing+\eta\cos2\overline{\varnothing})e^{-2\int(\xi\cos2\varnothing-\eta\sin2\varnothing)dx}dx\right]_t=0.$$

即，$\varepsilon'=4c(t)\varepsilon-h(t)$. 定理证毕。

Ⅱ　MKdV 方程的新解

在方程 (1) 和 (3) 之间存在一个变换

$$q=-(\varnothing_x+\xi\sin2\varnothing+\eta\cos2\varnothing)\qquad(14)$$

事实上，将 (14) 代入 (1)，我们有

$$q_t-(q_{xxx}+6q^2q_x)$$
$$=-(D_x+2\xi\cos2\varnothing-2\eta\sin2\varnothing)(\varnothing_t-\varnothing_{xxx}-2\varnothing_x^3-6\varnothing_x(\xi\sin2\varnothing+\eta\cos2\varnothing)^2).$$

由于当我们将 ($\varnothing$，ξ，η) 变为 ($-\varnothing$，$-\xi$，η) 时，方程 (3) 是不变的，因此在方程 (1) 和 (3) 之间存在另一个变换

$$q=\varnothing_x-\xi\sin2\varnothing-\eta\cos2\varnothing\qquad(15)$$

按照定理 3，我们可以由一般 CDF 方程的已知解得到新解。那么，由 (14) 和 (15)，我们可以得到 MKdV 方程 (1) 的对应新解

$$q_1=\pm(\overline{\varnothing}_x+\xi\sin2\overline{\varnothing}+\eta\cos2\overline{\varnothing}),$$
$$q_2=\pm(\overline{\varnothing}_x-\xi\sin2\overline{\varnothing}-\eta\cos2\overline{\varnothing}),$$

在 $\xi=0$ 的情况，我们可以得到 MKdV 方程 (1) 的一组单参数新解

例 1　我们取定方程 (3) 的平凡解 $\varnothing=0$ 和

$$\int\sin2\varnothing dx=0,\ \int\cos2\varnothing e^{2\eta\int\sin2\varnothing dx}dx=x.$$

那么 $c=0$，$h=12\eta^3$，$\varepsilon=-12\eta^3 t$，$Q=\alpha-2\eta x-12\eta^3 t$，$Y=1/Q$ (α是一个任意常数)，我们得到方程 (3) 的解 $\overline{\varnothing}=\arctan Y$ 和 MKdV 方程 (1) 的对应解：

$$q_1=\pm(\overline{\varnothing}_x+\eta\cos2\overline{\varnothing})=\pm\eta$$
$$q_2=\pm(\overline{\varnothing}_x-\eta\cos2\overline{\varnothing})=\pm\eta(3-Q^2)/(1+Q^2).$$

例 2　我们考虑方程 (3) 的平凡解 $\varnothing=\pi/4$，并取定

$$\int\cos2\varnothing dx=0,\ \int\sin2\varnothing e^{-2\xi\int\cos2\varnothing dx}dx=x.$$

那么 $c=0$，$h=-12\xi$。我们得到方程 (3) 的解 $\overline{\varnothing}=\arctan\left(1-\frac{1}{\omega}\right)$ ($\omega=\xi x+6\xi t+\delta$，$\delta$是一个任意常数) 和 MKdV 方程 (1) 的对应解：

$$q_1=\pm(\overline{\varnothing}_x+\xi\sin2\overline{\varnothing})=\pm\xi$$
$$q_2=\pm(\overline{\varnothing}_x-\xi\sin2\overline{\varnothing})=\pm\left(1-\frac{1}{H}\right),$$
$$\left(H=\omega^2-\omega+\frac{1}{2}\right).$$

在 $\xi\neq0$ 和 $\eta\neq0$ 的情况，我们可以得到 MKdV 方程 (1) 的一组双参数新解。

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REIationship Between Retention And HomologousSeauence Number of Long Chain Alcohol Acid And Ester in Rplc

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Abstract Based on molecular structure and Jiang Mingaian′s organic compound homologous linear rule, the quantitative relationship between reten tion and the homologous sequ en ce number is identifed for long Chain alcohool, acid and ester in reversed－phase high－performanceliquig Chromatography (RPLLC) and is supported by experimental data. The result is Satisffactory.

Keywords Reversed－phase liquid Chromatography (RPLC) Retention Homologous Sequence number.

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A Constant Property of general CDF equation

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Abstract In this paper. We give a constant property of general CDF equation. According to it, We may get a new Solution from known Solution of a general CDF equation, only using integral method.

Keywords CDF equation Backlund transform Mkdv equation